LOGICAL COMPLEXITY AND COGNITIVE DIFFICULTY IN REASONING

Jakub Szymanik

WARM-UP

BALL & BAT (KAHNEMAN & FRANK)

- A ball and a bat together cost $1.10.
- The bat costs $1 more than the ball.
- How much does the ball cost?

AN EXAMPLE FROM HECTOR LEVESQUE

Jack is looking at Anne, but Anne is looking at George. Jack is married, but George is not. Is a married person looking at an unmarried person?

(a) Yes

(b) No

(c) Not enough information to determine

LINGUISTIC REASONING

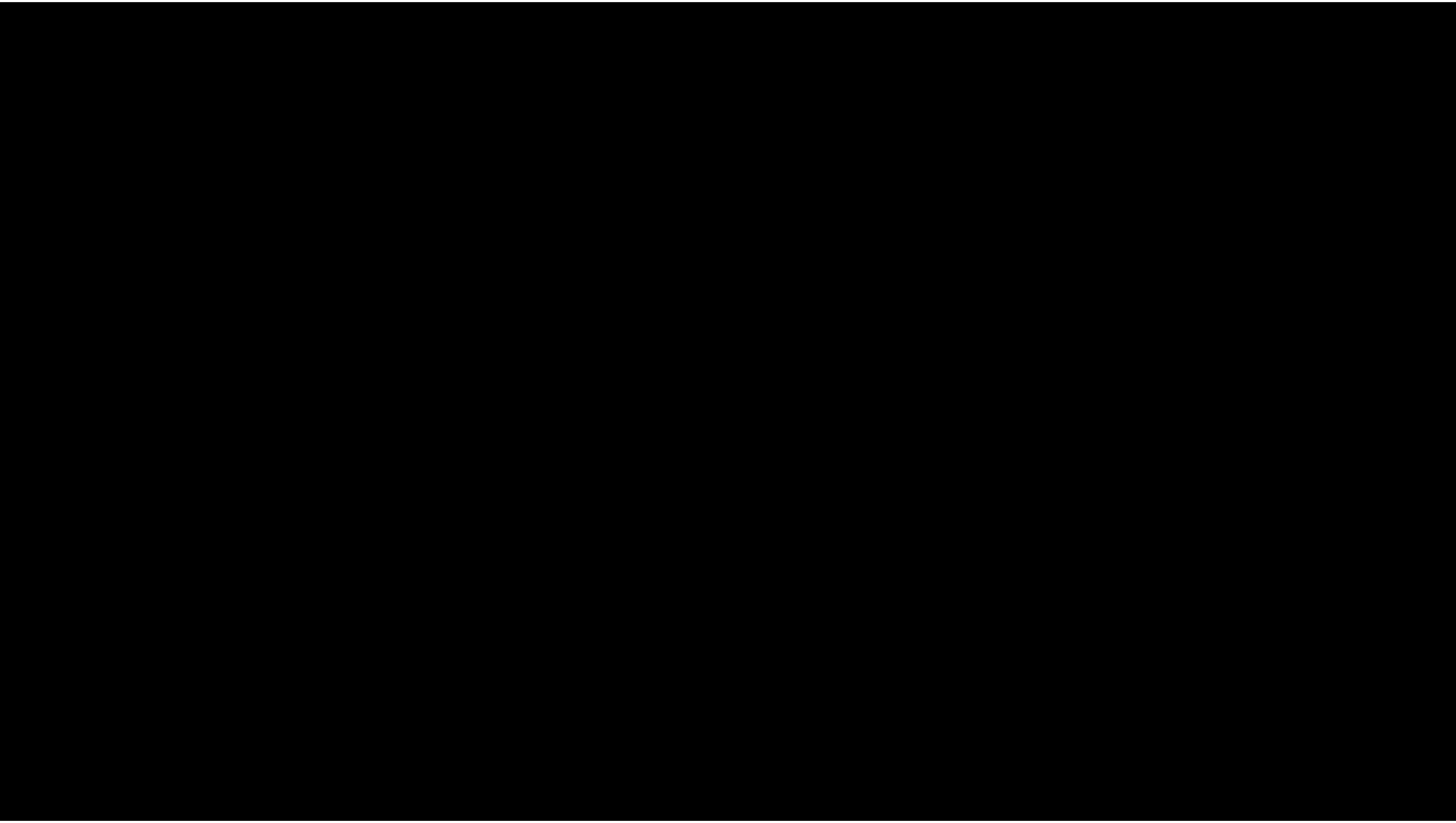
Some zookeepers are pacifists

No pacifists are troglodytes

Some zookeepers are not troglodytes

MORE RIDDLES HERE

MORE RIDDLES HERE



"

The sole end of logic is to explain the principles and operations of our reasoning faculty.

-David Hume

INTRODUCTION

LOGICAL MODELING

- Sometimes also called Declarative/Symbolic modeling.
- Goal: To systematize (parts) of cognition on the concept of logical system and the notion of reasoning and computing in such systems.
- The oldest paradigm for modeling the mind (since Aristotle)
- Good at: finding certain intrinsic (context-independent, combinatorial) structures that predict and explain human thinking.
- Top-down information processing systems, often closely link with AI.

DIVIDE BETWEEN LOGIC AND PSYCHOLOGY

- Kant: logical laws as the fabric of thoughts
- 19th century: logic=psychologism (Mill)
- Frege’s anti-psychologism enforced separation
- 19/20th century beginnings of modern logic and psychology
- ’60 witness the growth of cognitive science but also semantic and computational turn in logics pivoting around the notion of interpretation and processing.

EXAMPLE: THE WASON SELECTION TASK

If there is a D on one side of the card, then there is a 3 on the other.

	D	F	3	7
Experimental Results	89%	16%	62%	25%

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	beer	coke	22yrs	16yrs
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Reasoning `to’ and `from’ the interpretation (van Lambalgen and Stenning, 2008)

LEVELS OF INFORMATION PROCESSING

1. Computational level: specify cognitive task
2. Algorithmic level: the algorithms that may be used
3. Implementation level: how this is actually done in the brain

Marr'83

HOW LOGIC CONTRIBUTES?

- Helps to rigorously formulate `problems’
- Logic informs about intrinsic properties of a problem
- Structural properties correlate with human performance
- Logic captures inherent cognitive complexity

EXPLORING RECENT CASE STUDIES

- Categorization
- Syllogistic reasoning
- Processing meaning
- Reasoning about others
- Strategic reasoning
- Problem solving
- …

TOPIC 0: BOOLEAN CATEGORIZATION

BOOLEAN RELATIONS

- Boolean relations are a way to create new concepts:

  `cousin' is a child of an uncle or aunt

  'beer' is an alcoholic beverage usually made from malted cereal grain and flavored with hops, and brewed by slow fermentation

  in basketball, `travel' is illegally moving the pivot foot or taking three or more steps without dribbling

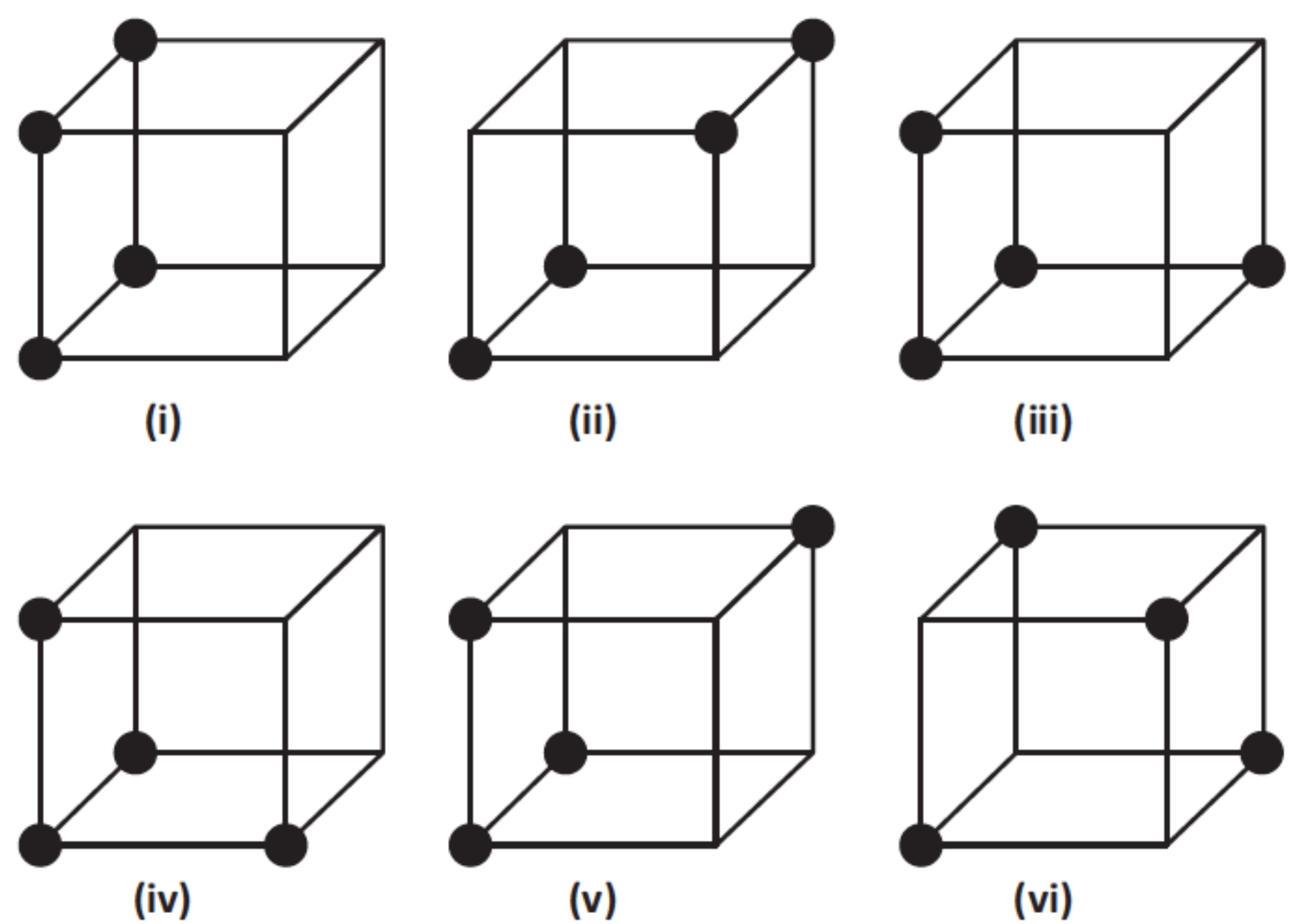
  `depression' is a mood disorder characterized by persistent sadness and anxiety, or feeling of hopelessness and pessimism, or …

QUESTIONS

- How people acquire, represent, and use concepts?
- E.g., concepts depending on and are easier to learn than those depending on or (Bruner et al. `65).
- But the data seems more puzzling (see next slide).
- What's the logical theory of complexity here?

SHEPARD TREND

- Six different sorts of concept based on three binary variables
- Each concept: 4 instances and 4 non-instances in 8 possibilities
- Different presentations methods: sequentially, simultaneously, etc.
- Dependent variables: errors, latencies, accuracy of descriptions, etc.
- I < II < III, IV, V < VI



Shepard et al.'61

THE INSTANCES OF THE CONCEPTS

Concept number	Instances
I	not-a b c not-a b not-c not-a not-b c not-a not-b not-c
II	a b c a b not-c not-a not-b c not-a not-b not-c
III	a not-b c not-a b not-c not-a not-b c not-a not-b not-c
IV	a not-b not-c not-a b not-c not-a not-b c not-a not-b not-c
V	a b c not-a b not-c not-a not-b c not-a not-b not-c
VI	a b not-c a not-b c not-a b c not-a not-b not-c

BOOLEAN COMPLEXITY

- The length of the shortest Boolean formula logically equivalent to the concept, e.g., expressed in terms of the number of literals (positive or negative variables).
- ~ Intrinsic mathematical complexity of the concept.
- ~ Kolmogorov complexity of \`incompressibility’.
- Btw, finding the shortest formula is intractable.
- *(a and b) or (a and not b) or (not a and b)* reduces to *(a or b)*

BOOLEAN COMPLEXITY AND DATASET

- Minimal description predicts learning difficulty (Feldman ’01).
- But *(a and b) < (a or b)*
- So parity assumption: concepts with fewer instances than non-instances should be easier to learn than those with fewer non-instance than instances.

CAPTURES SHEPARD TREND

Concept number	Instances	Minimal description
I	not-a b c not-a b not-c not-a not-b c not-a not-b not-c	*not a* (1)
II	a b c a b not-c not-a not-b c not-a not-b not-c	*(a and b) or (not a and not b)* (4)
III	a not-b c not-a b not-c not-a not-b c not-a not-b not-c	*(not a and not c) or (not b and c)* (4)
IV	a not-b not-c not-a b not-c not-a not-b c not-a not-b not-c	*(not c or (not a and not b)) and (not a or not b)* (5)
V	a b c not-a b not-c not-a not-b c not-a not-b not-c	*(not a and not (b and c)) or (a and (b and c))* (6)
VI	a b not-c a not-b c not-a b c not-a not-b not-c	(a and ((not b and c) or (b and not c))) or (not a and ((not b and not c) or (b and c))) (10)

NEW DATA SET

- Consider an arbitrary Boolean concept defined by P positive examples over D binary features.
- For Shepard types D=3 and P=4.
- Feldman studies 76 Boolean concepts.

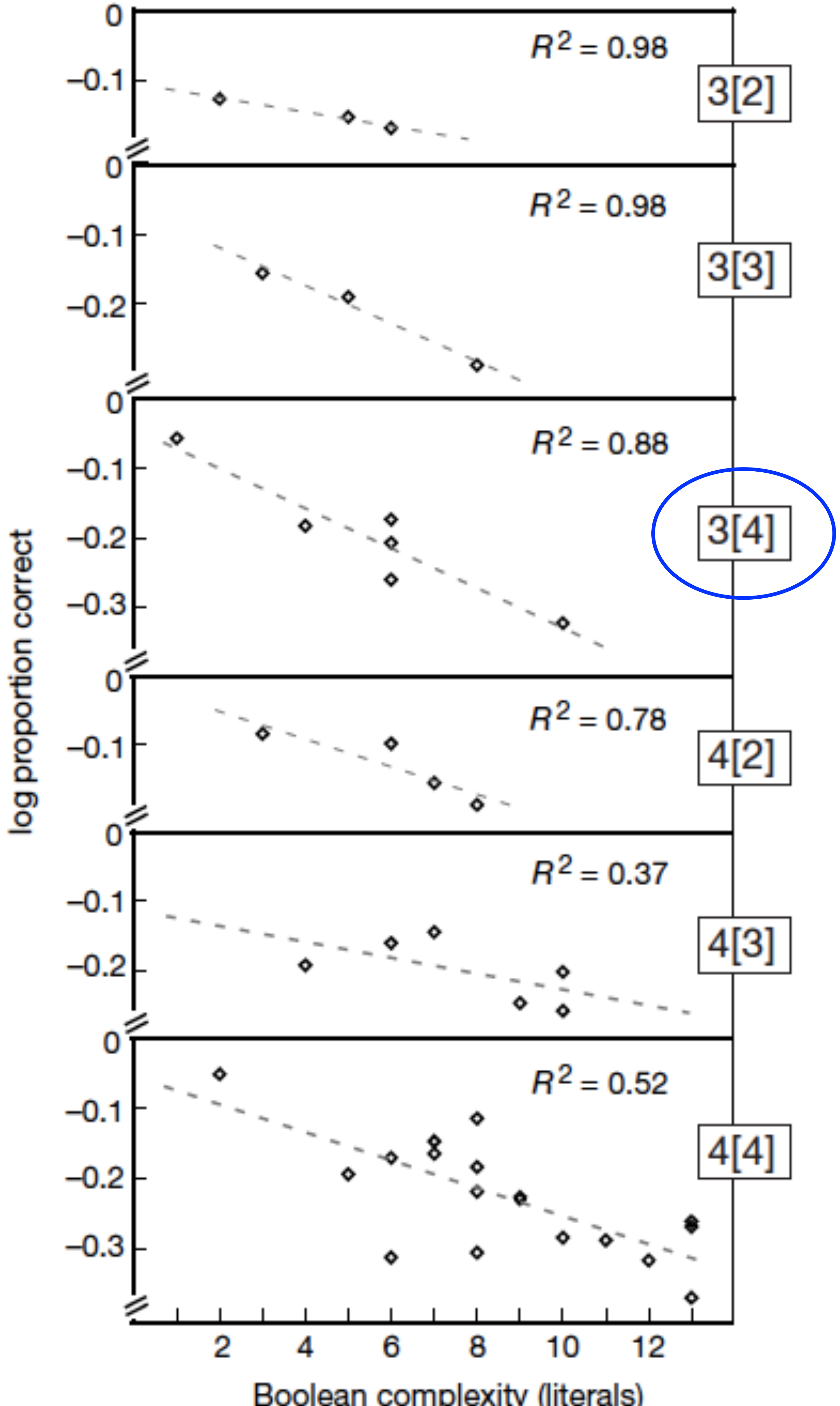
P

$\lvert D[P] \rvert$	1	2	3	4	5
D 1	1				
2	1	2			
3	1	**3**	**3**	**6**	
4	1	**4**	**6**	**19**	27
5	1	5			

Tested families

SHJ family

Feldman `01



RESULTS

- Boolean complexity accounts for 50% of variance in the dataset.

QUESTIONS

- Which Boolean connectives?
- Constructing minimal descriptions is intractable.
- Parity itself explains 20% of variance.
- So, a recent flurry of alternative models: Feldman `06, Vigo `09, Goodwin et al. `13,…

BOOLEAN LANGUAGE COMPARISON

- Bayesian concept learning model (Goodman et al., `08).
- Bayesian data analysis model: which representational system is the most likely, given human responses?

Grammar	H.O.LL	FP	$R^2_{response}$	R^2_{mean}
FullBoolean	−16296.84	27	.88	.60
Biconditional	−16305.13	26	.88	.64
CNF	−16332.39	26	.89	.69
DNF	−16343.87	26	.89	.66
SimpleBoolean	−16426.91	25	.87	.70
Implies	−16441.29	26	.87	.70
HornClause	−16481.90	27	.87	.65
NAND	−16815.60	24	.84	.61
NOR	−16859.75	24	.85	.58
uniform	−19121.65	4	.77	.06
exemplar	−23634.46	5	.55	.15
OnlyFeatures	−31670.71	19	.54	.14
Response-Biased	−37912.52	4	.03	.04

Piantadosi et al. `16

TOPIC 1: SYLLOGISTIC REASONING

Logic

explanation

Probability

prediction

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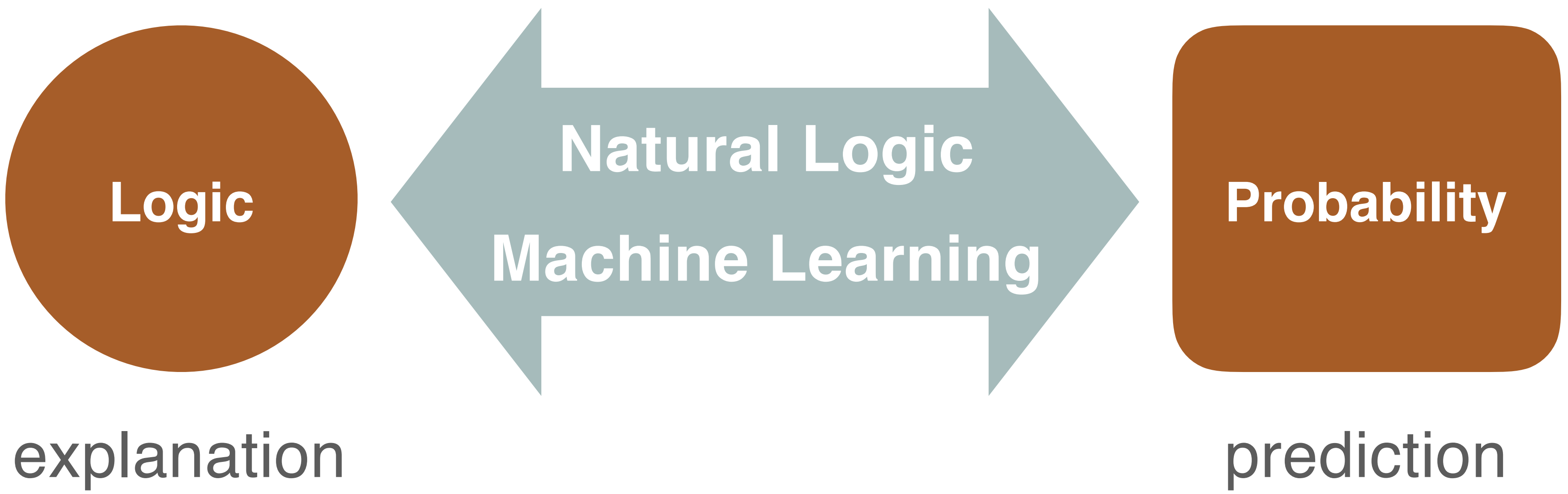
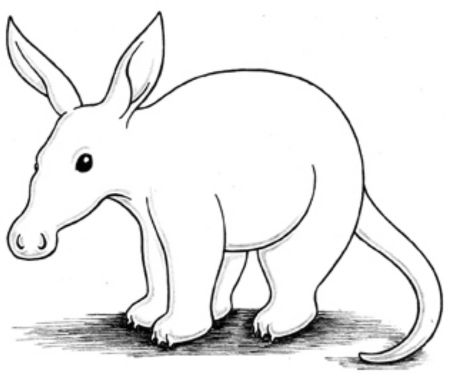


CASE STUDY



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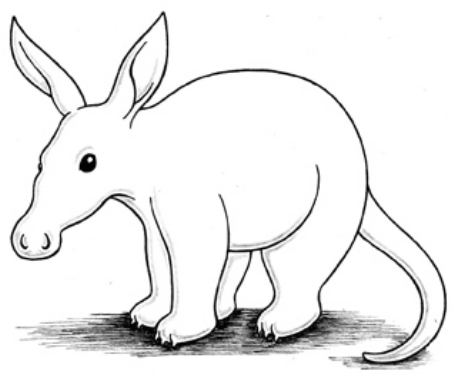

1. *All aardvarks are insectivores.*

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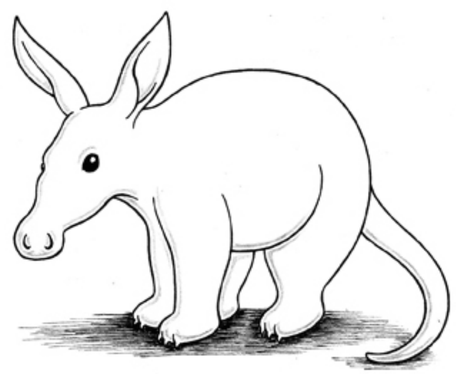

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2. *All Orycteropodidae are aardvarks.*

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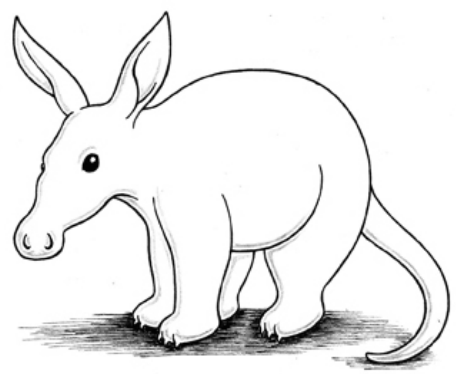

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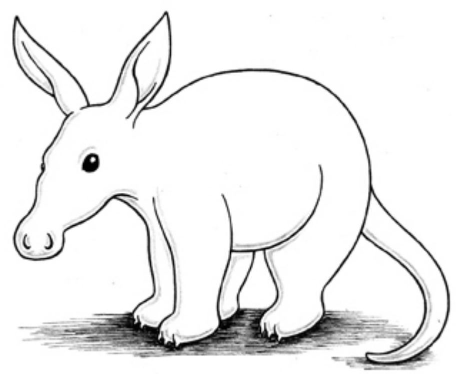

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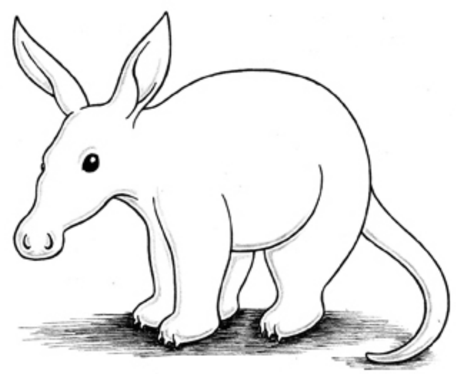

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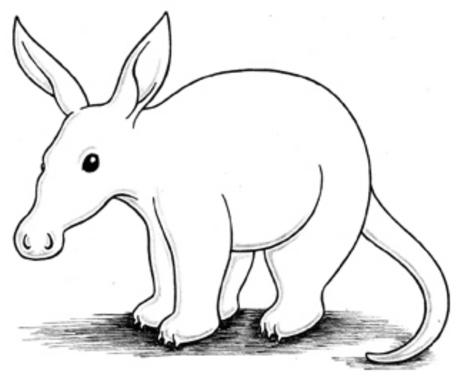

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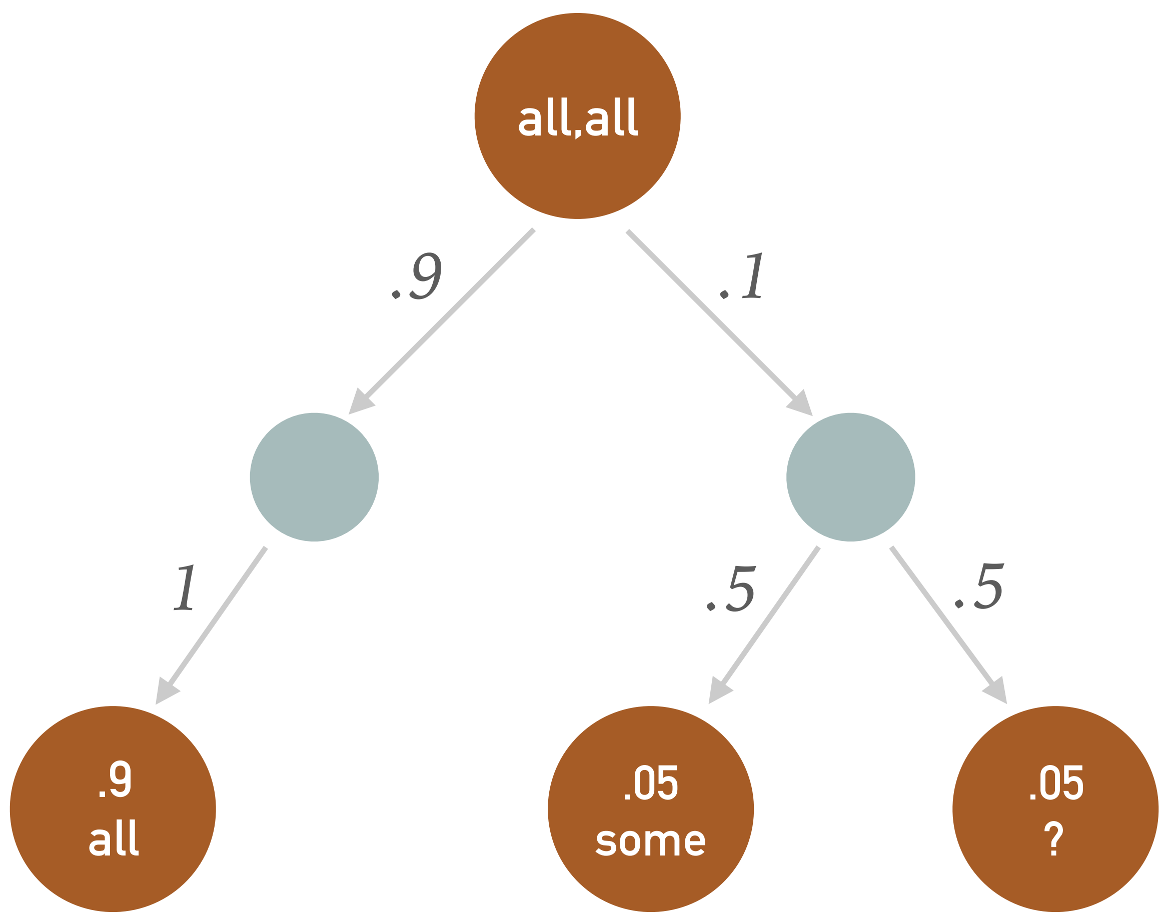
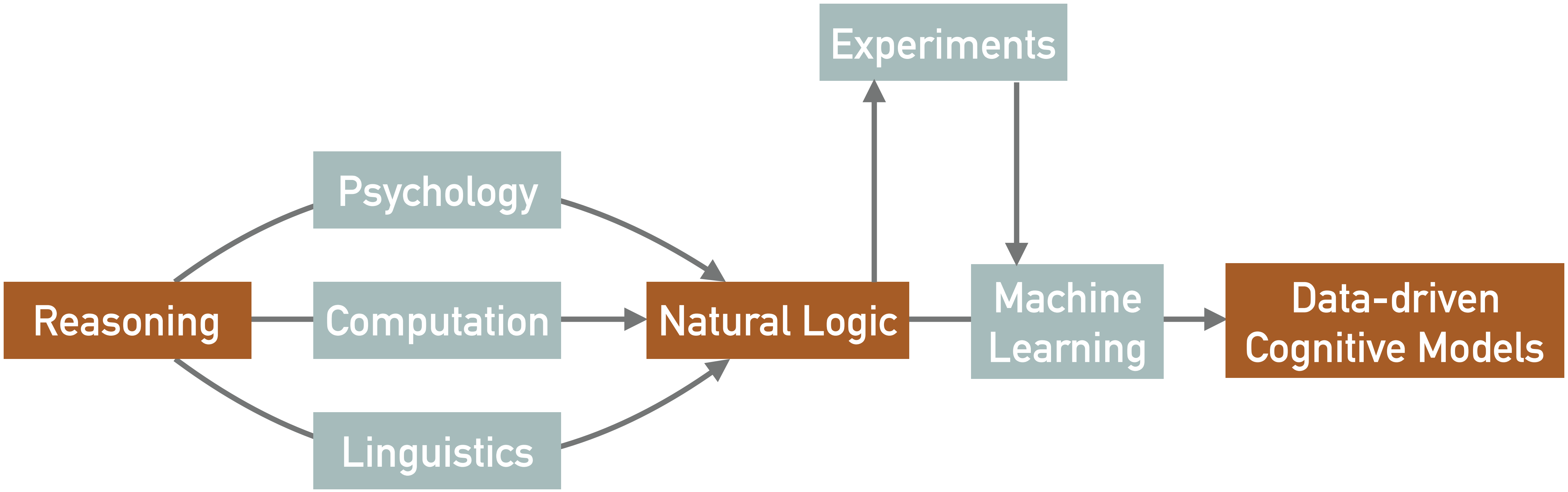


PLAN OF ACTION



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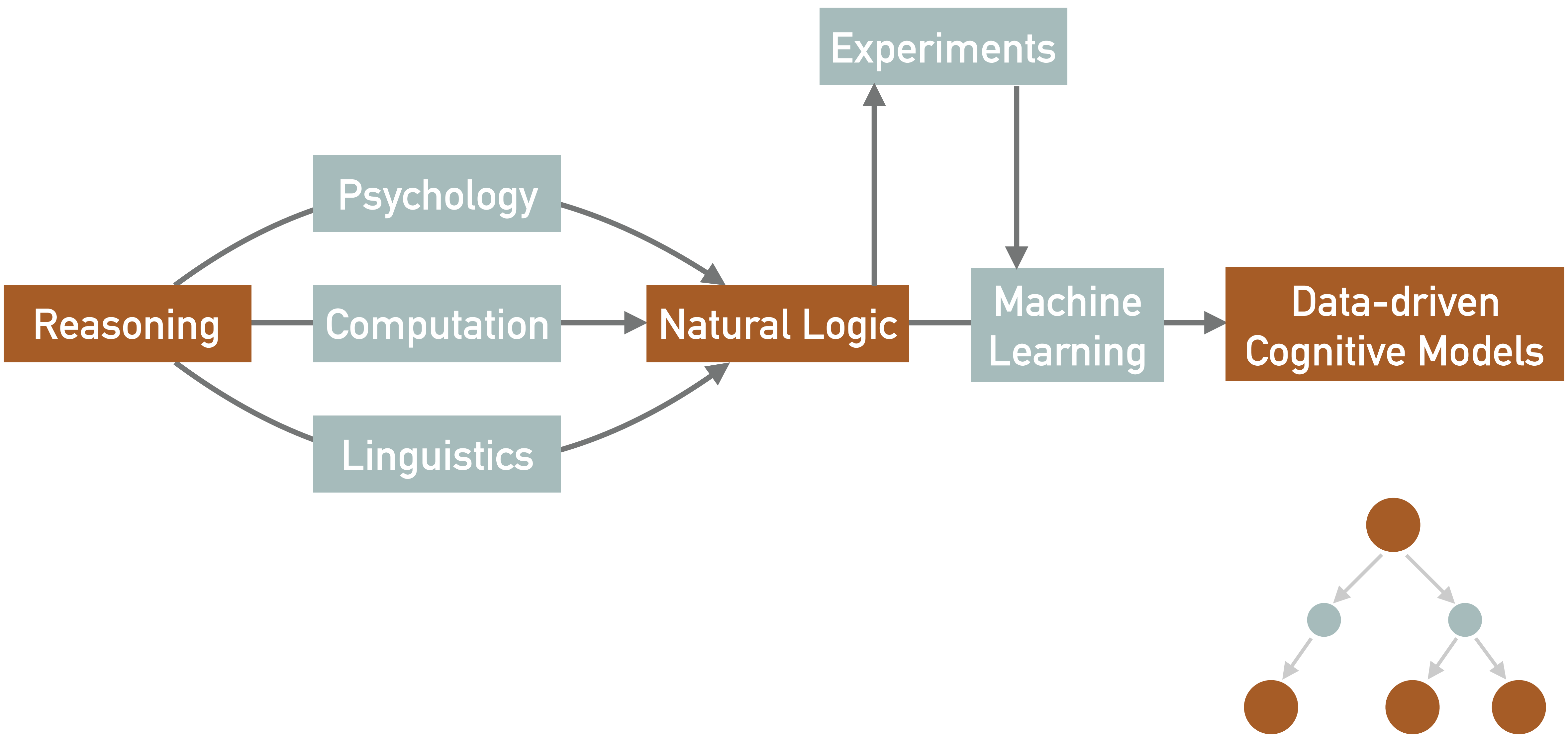
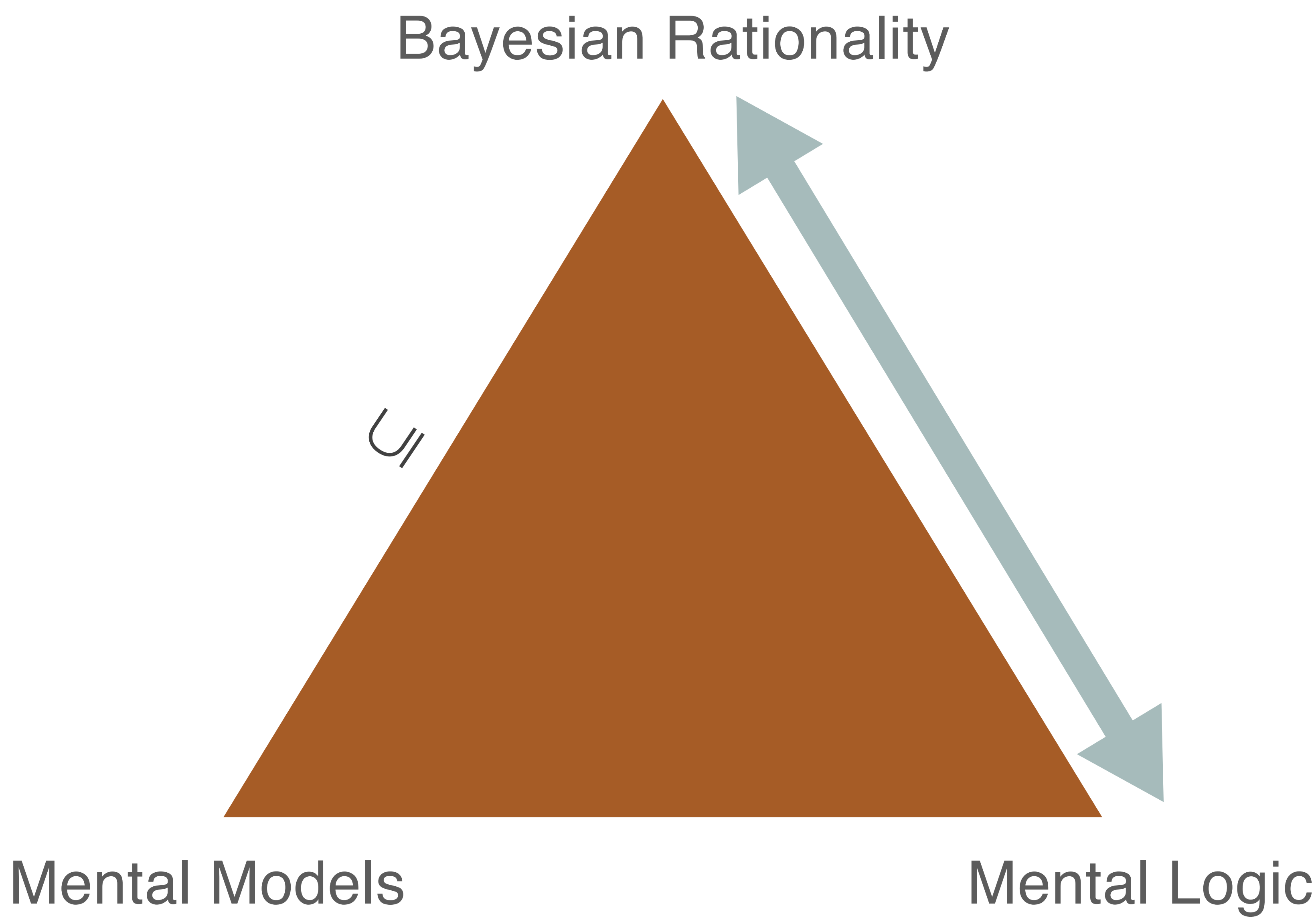


PSYCHOLOGICAL THEORIES OF REASONING



MENTAL LOGIC

- Rips (1994):
- Formulas as the underlying mental representations
- Inference rules are the basic operations
- PSYCOP based on Natural Deduction
- You can think about proofs as computations

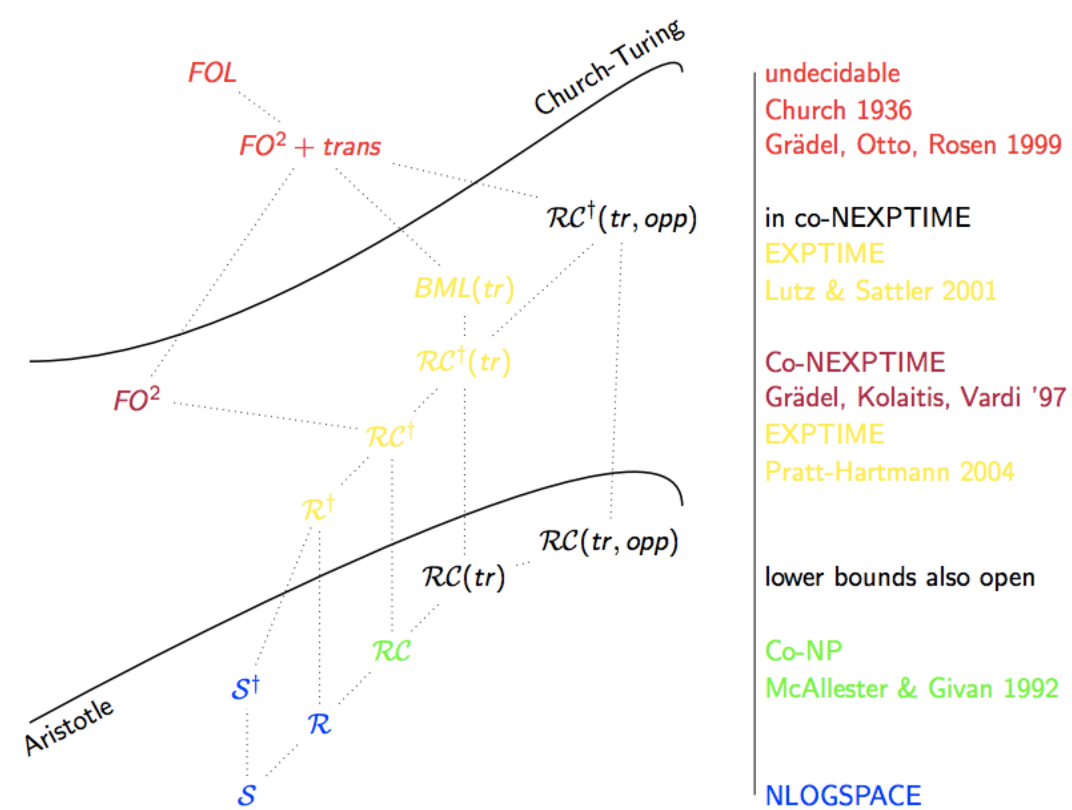
MENTAL LOGIC'S SHORTCOMINGS

- Abstract rules and formal representations
- Based in natural deduction for FOL
- Ad hoc `psychological completness'
- Explains only validities, no story on mistakes
- No learning or individual differences

QUICK FIX: NATURAL LOGIC PROGRAM

using linguistic constructs directly as vehicles of inference

- van Benthem 1986, Sánchez-Valencia 1991:
- They are natural!
  - All aardvarks are insectivores.
  - $\forall x[Aardvark(x) \implies Insectivore(x)]$
  - All (Aardvarks, Insectivores)
- They scale up!



Pratt-Hartmann'04

BENCHMARK TASK: SYLLOGISTICS

- All A are B : universal affirmative (A)
- Some A are B: particular affirmative (I)
- No A are B: universal negative (E)
- Some A are not B: particular negative (O)

Figure 1:	Figure 2:	Figure 3:	Figure 4:
BC	CB	BC	CB
<u>AB</u>	<u>AB</u>	<u>BA</u>	<u>BA</u>
AC	AC	AC	AC

EA2E:

No C are B
All A are B
No A are C

VALID REASONING

- An argument is valid if and only if it takes a form that makes it impossible for the premises to be true and the conclusion nevertheless to be false.
- True under every interpretation.

All men are mortal.

Socrates is a man.

Therefore, Socrates is mortal.

Some men are famous.

Socrates is a man.

Therefore, Socrates is famous.

DATA - SYLLOGISTIC REASONING

premisses	*conclusion*				*premisses*	*conclusion*				*premisses*	*conclusion*			
& figure	A	I	E	O	*& figure*	A	I	E	O	*& figure*	A	I	E	O
AA1	90	5	0	0	AO1	1	6	1	57	IO1	3	4	1	30
AA2	58	8	1	1	AO2	0	6	3	67	IO2	1	5	4	37
AA3	57	29	0	0	AO3	0	10	0	66	IO3	0	9	1	29
AA4	75	16	1	1	AO4	0	5	3	72	IO4	0	5	1	44
AI1	0	92	3	3	OA1	0	3	3	68	OI1	4	6	0	35
AI2	0	57	3	11	OA2	0	11	5	56	OI2	0	8	3	35
AI3	1	89	1	3	OA3	0	15	3	69	OI3	1	9	1	31
AI4	0	71	0	1	OA4	1	3	6	27	OI4	3	8	2	29
IA1	0	72	0	6	II1	0	41	3	4	EE1	0	1	34	1
IA2	13	49	3	12	II2	1	42	3	3	EE2	3	3	14	3
IA3	2	85	1	4	II3	0	24	3	1	EE3	0	0	18	3
IA4	0	91	1	1	II4	0	42	0	1	EE4	0	3	31	1
AE1	0	3	59	6	IE1	1	1	22	16	EO1	1	8	8	23
AE2	0	0	88	1	IE2	0	0	39	30	EO2	0	13	7	11
AE3	0	1	61	13	IE3	0	1	30	33	EO3	0	0	9	28
AE4	0	3	87	2	IE4	0	1	28	44	EO4	0	5	8	12
EA1	0	1	87	3	EI1	0	5	15	66	OE1	1	0	14	5
EA2	0	0	89	3	EI2	1	1	21	52	OE2	0	8	11	16
EA3	0	0	64	22	EI3	0	6	15	48	OE3	0	5	12	18
EA4	1	3	61	8	EI4	0	2	32	27	OE4	0	19	9	14
										OO1	1	8	1	22
										OO2	0	16	5	10
										OO3	1	6	0	15
										OO4	1	4	1	25

A = all E = no
I = some O = some … not

Chater and Oaksford'99

GEURT'S 2003 MODEL

- Logic including syllogistics and pivoting on monotonicity:
- All-Some: `All A are B' implies `Some A are B'.
- No-Some not: `No A are B' implies `Some A are not B'.
- Conversion1: `Some A are B' implies `Some B are A';
- Conversion2: `No A are B' implies `No B are A''.
- Monotonicity: If A entails B, then the A in any upward entailing position can be substituted by a B, and the B in any downward entailing position can be substituted by an A.
- Extra rule: `No A are B' and `Some C are A' implies `Some C are not B'.

MONOTONICITY EXPLAINED

- Some boy is dirty so Some child is dirty. **(upward)**
- All children are dirty so All boys are dirty. **(downward)**
- **Some not? No?**

EXAMPLE OF A SYLLOGISTIC PROOF

No C are B	*(1)*	
All A are B	*(2)*	
No B are C	*(3)*	*Conversion (1)*
No A are C	*(4)*	*Monotonicity (2,3)*

INHERENT COMPLEXITY

- The shorter the proof the easier the syllogism.
- Initial budget of 100 units. Each use of the monotonicity rule costs 20, the extra rule costs 30; a proof containing a "Some Not" proposition costs an additional 10 units. Take the remaining budget as an evaluation of the difficulty.
- It gives a good fit with data.

AA1A	80	(90)	OA3O	70	(69)	EA1O	40	(3)
EA1E	80	(87)	AO2O	70	(67)	EA2O	40	(3)
EA2E	80	(89)	EI1O	60	(66)	EA3O	40	(22)
AE2E	80	(88)	EI2O	60	(52)	EA4O	40	(8)
AE4E	80	(87)	EI3O	60	(48)	AE2O	40	(1)
IA3I	80	(85)	EI4O	60	(27)	AE4O	40	(2)
IA4I	80	(91)	AA1I	60	(5)			
AI1I	80	(92)	AA3I	60	(29)			
AI3I	80	(89)	AA4I	60	(16)			

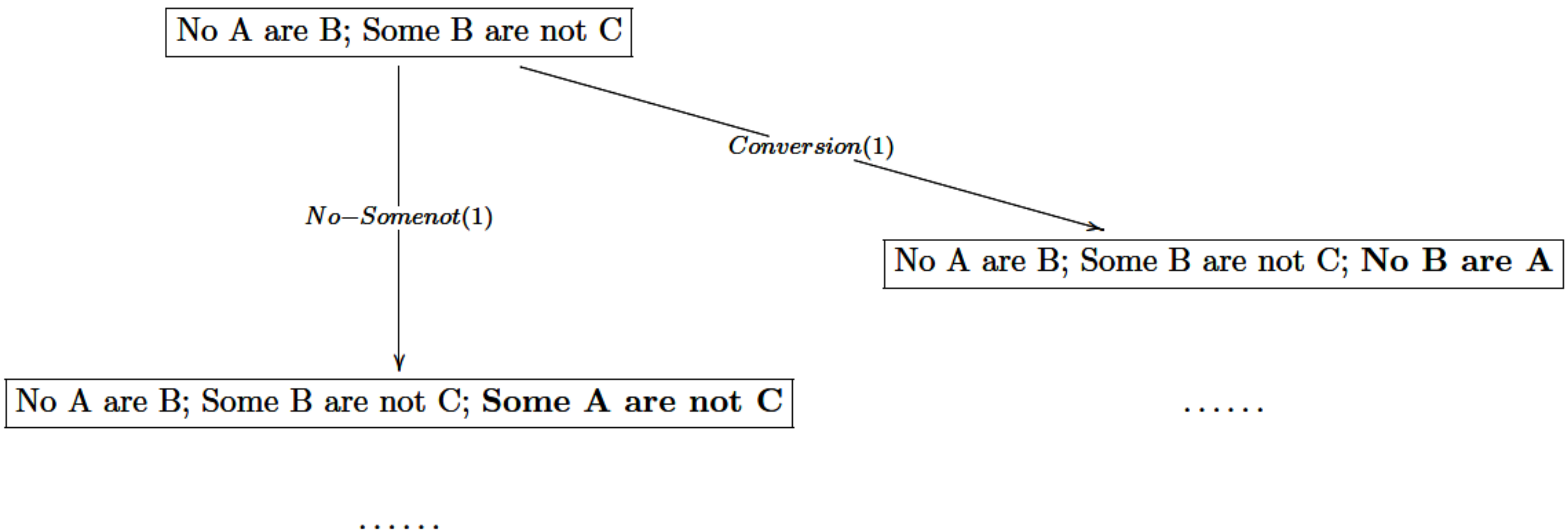
Predicted difficulty and data

Geurts'03

SHORT-COMINGS OF GEURTS' APPROACH

- Arbitrary set of rules
- Arbitrary weights
- But we can learn these from the data

PROBABILISTIC INFERENCE



- Geurts' logic
- Tree representation: states linked by reasoning events
- No vapid transitions

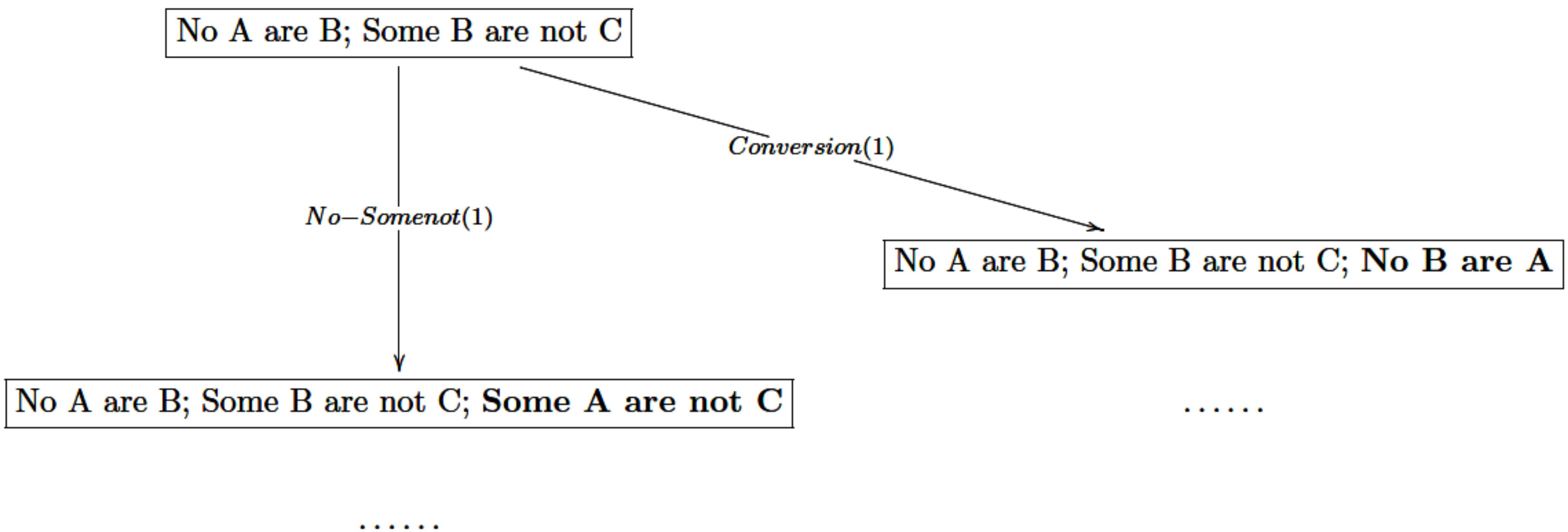
PROBABILITIES

- Tendency value, w_r: an `easier 'rule is adopted with higher probability, while a more difficult one is adopted with lower probability.

$$p(r|S, \mathbf{w}) = \frac{w_r}{w_G + \sum_{r' \in R} c_{r'} \cdot w_{r'}}$$

- w_r is a weight estimated (for every rule r) from the data
- c_r the number of ways that rule r can be adopted at S
- w_G reserves probability mass for `terminating' the inference at state S and making a heuristic guess

EXAMPLE: OE1



$$p(r_l|S, w_{r_l} w_{r_r}) = \frac{w_{r_l}}{w_G + 1w_{r_l} + 1w_{r_r}}$$

THE OUTPUT OF THE MODEL

- A probability with which a syllogism is endorsed.
- 5 possible conclusions: A, I, E, O, NVC.
- We model transition probabilities.
- We compute the probability that a given conclusion is drawn.

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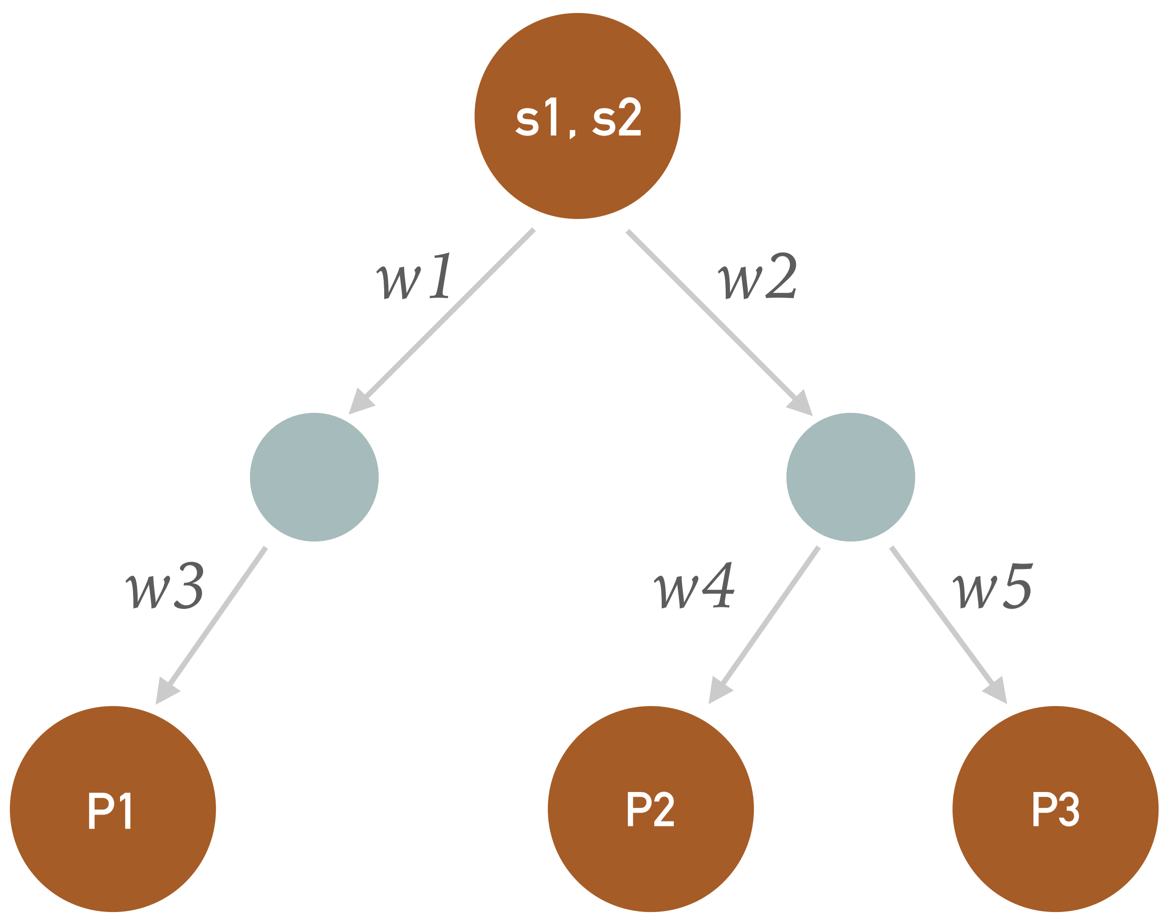


TRAINING

- Subset of the data from Chater and Oaksford (1999).
- We use the generalized expectation maximization: there is no closed-form solution for the M step.
- Compute:

$$\arg\max_{\theta} \; p(\{(X_i, y_i)\}_{i \leq n} | \theta)$$

EVALUATION

- The Khemlani and Johnson-Laird (2012) method.
- Detection theory.
- They assume there is a lot of random noise in the data.

Predictions \ Exp. Data	$< 30\%$	$\geq 30\%$
$< 30\%$	Correct Rejection	Miss
$\geq 30\%$	False Alarm	Hit

PERFORMANCE

- 95,8% correct predictions on syllogisms
  with at least one conclusion.
- 81,6% correct predictions on all syllogisms.
- But no mechanism to explain the errors.
- The models always returns NVC for invalid syllogisms.

Zhai et al.'15

GOING BOTTOM-UP: ILLICIT CONVERSION

- Conversion: For every Q,

  `Q A are B' implies `Q B are A',
- This extension halves the number of misses.
- 91,9% correct predictions on all syllogisms.

UNCERTAINTY AND ERRORS

- Probability of guessing `nothing follows' is negatively related to the informativeness of the premises
- Atmosphere hypothesis:
  A. when there is a negation in the premises, subjects are likely to draw a negative conclusion
  B. when there is `some' in the premises it will be likely in the conclusion
  C. when neither is the case, the conclusion is often affirmative

PERFORMANCE OF THE FULL MODEL

- 95% correct predictions on all syllogisms
- The training gives the informativeness order as assumed by Chater & Oaksford:

A> E > I > O

- And data yields the complexity order:

Conversion<Monotonicity<All-Some<No-SomeNot

Zhai et al.'15

COMPARING WITH OTHER THEORIES

Khemlani and Johnson-Laird (2012)

Theory	Hit	Miss	False Alarm	Correct Rejection	Correct Predictions
Atmosphere	44	41	20	215	259 /80.9%
Matching	41	44	55	180	221 /69.1%
Conversion	52	33	12	223	275 /85.9%
PHM*	40	45	63	172	212 /66.3%
PSYCOP	45	40	26	209	254 /79.4%
Verbal Models*	54	31	29	206	260 /81.2%
Mental Models*	85	0	55	180	265 /82.8%
Generative Model Ver. 1	51	33	26	210	261/81.6%
Generative Model Ver. 2	67	17	9	227	294/91.9%
Generative Model Ver. 3	74	10	6	230	304/95.0%
Experimental Data	85	0	0	235	320/100%

Zhai et al.'15

SUMMARY

- Deriving `psychological completeness' from data.
- Some rules are unlikely to fire.
- A way to classify inferences steps wrt difficulty/preferability.
- Yields computationally friendlier systems.
- Modular approach.

SOME FURTHER WORK

- Extend to wider fragments of language.
- Run experiments/train model on better data.
- Think about arising logics and proof systems.
- Think about processing model and its complexity.
- Pick natural reasoning rules (logic) from the data.
- Think about nonlinguistic tasks.
- …

TOPIC 1.1 WHAT WITH COMPLEX PATTERNS

Some of the sopranos sang with fewer than three of the tenors.

All sopranos were Italian.

Some of the sopranos sang with fewer than three of the Italians.

MONOTONICITY & DIFFICULTY

1. Some of the sopranos sang with more than three of the tenors.
2. None of the sopranos sang with fewer than three of the tenors.
3. Some of the sopranos sang with fewer than three of the tenors.

$$\frac{\begin{array}{c}\mathrm{Q}_1 A \text{ sang with } \mathrm{Q}_2 B \\ \text{All B were C./All C were B.}\end{array}}{\mathrm{Q}_1 A \text{ sang with } \mathrm{Q}_2 C}$$

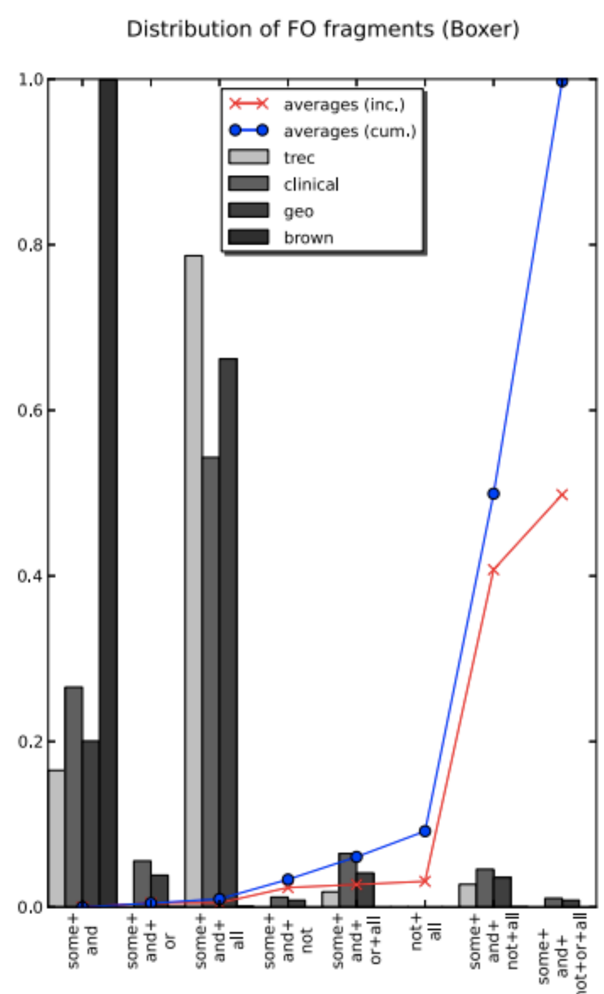
$$\uparrow \mathrm{Q}_1 \uparrow \mathrm{Q}_2 < \begin{array}{c} \uparrow \mathrm{Q}_1 \downarrow \mathrm{Q}_2 \\ \downarrow \mathrm{Q}_1 \uparrow \mathrm{Q}_2 \\ \downarrow \mathrm{Q}_1 \downarrow \mathrm{Q}_2 \end{array} \qquad \uparrow \mathrm{Q}_1 \uparrow \mathrm{Q}_2 < \downarrow \mathrm{Q}_1 \downarrow \mathrm{Q}_2 < \begin{array}{c} \uparrow \mathrm{Q}_1 \downarrow \mathrm{Q}_2 \\ \downarrow \mathrm{Q}_1 \uparrow \mathrm{Q}_2 \end{array}$$

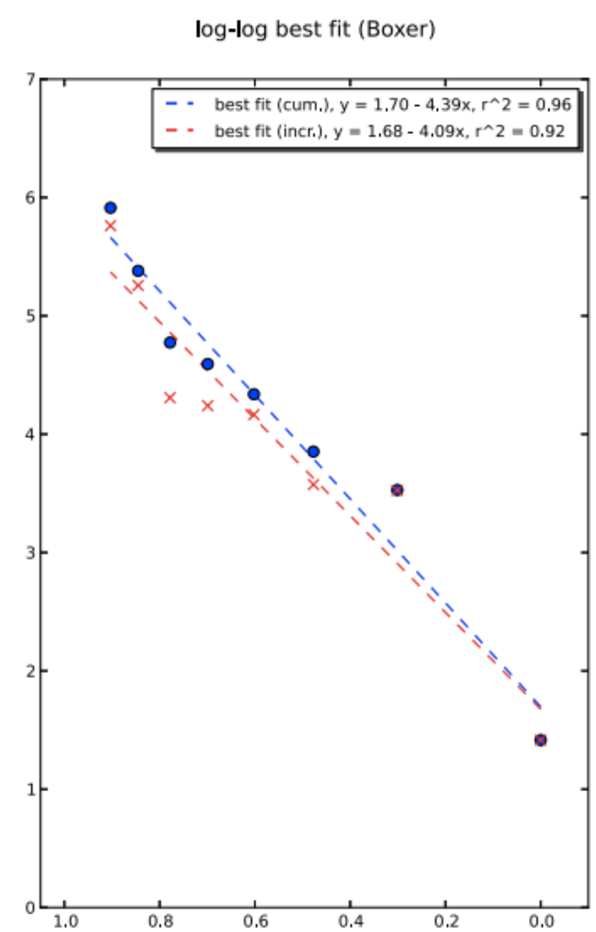
Geurts & Slik'05

Question: Can you ground it in a Natural Logic?

COMPLEXITY OF REASONING

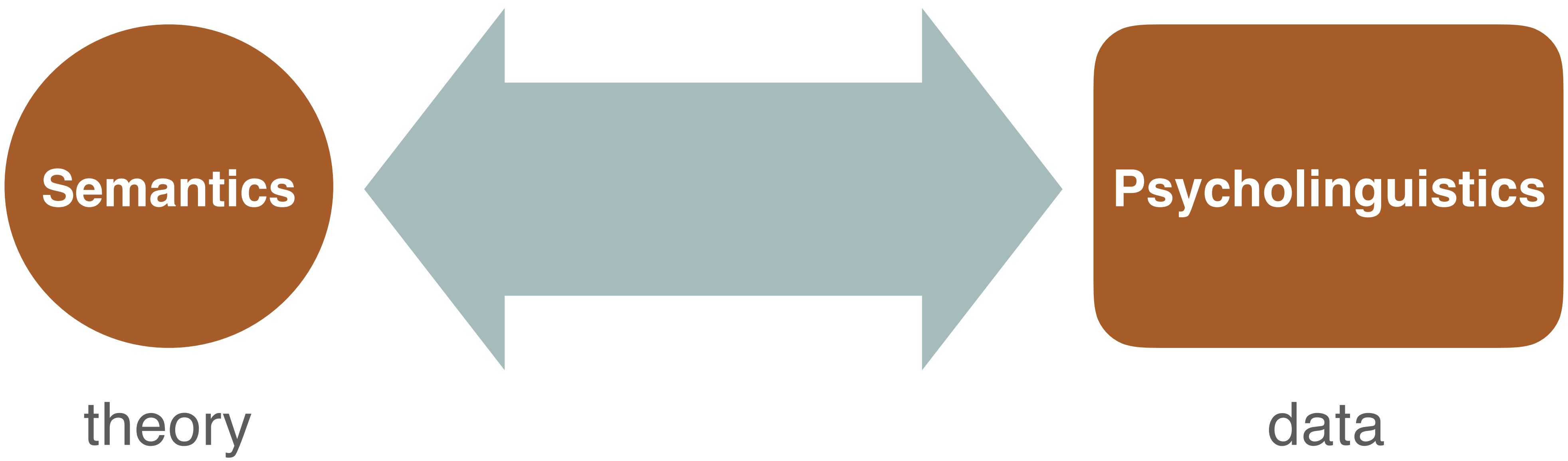
- How complex are natural language arguments?
- It depends on the underlying natural logic.
- Speakers tend to use “simple" messages.
- Semantic complexity correlates with linguistic frequency (Thorne, 2012)





Thorne’12

TOPIC 2: MEANING & COMPLEXITY



FORMAL SEMANTICS

How do we understand language?

FORMAL SEMANTICS

How do we understand language?

- Formal semantics builds precise models of meaning
- Success story in the last 50 years (language technology)
- E.g. explaining correctness (syntax not enough)

1. *There are many semantics textbooks.*
2. *There are most semantics textbooks. (*)*

Partee & ter Meulen'90, Kamp & Reyle'93, Portner'05, Winter'16, Dekker & Aloni'16

PSYCHOLINGUISTICS

How do we understand language?

PSYCHOLINGUISTICS

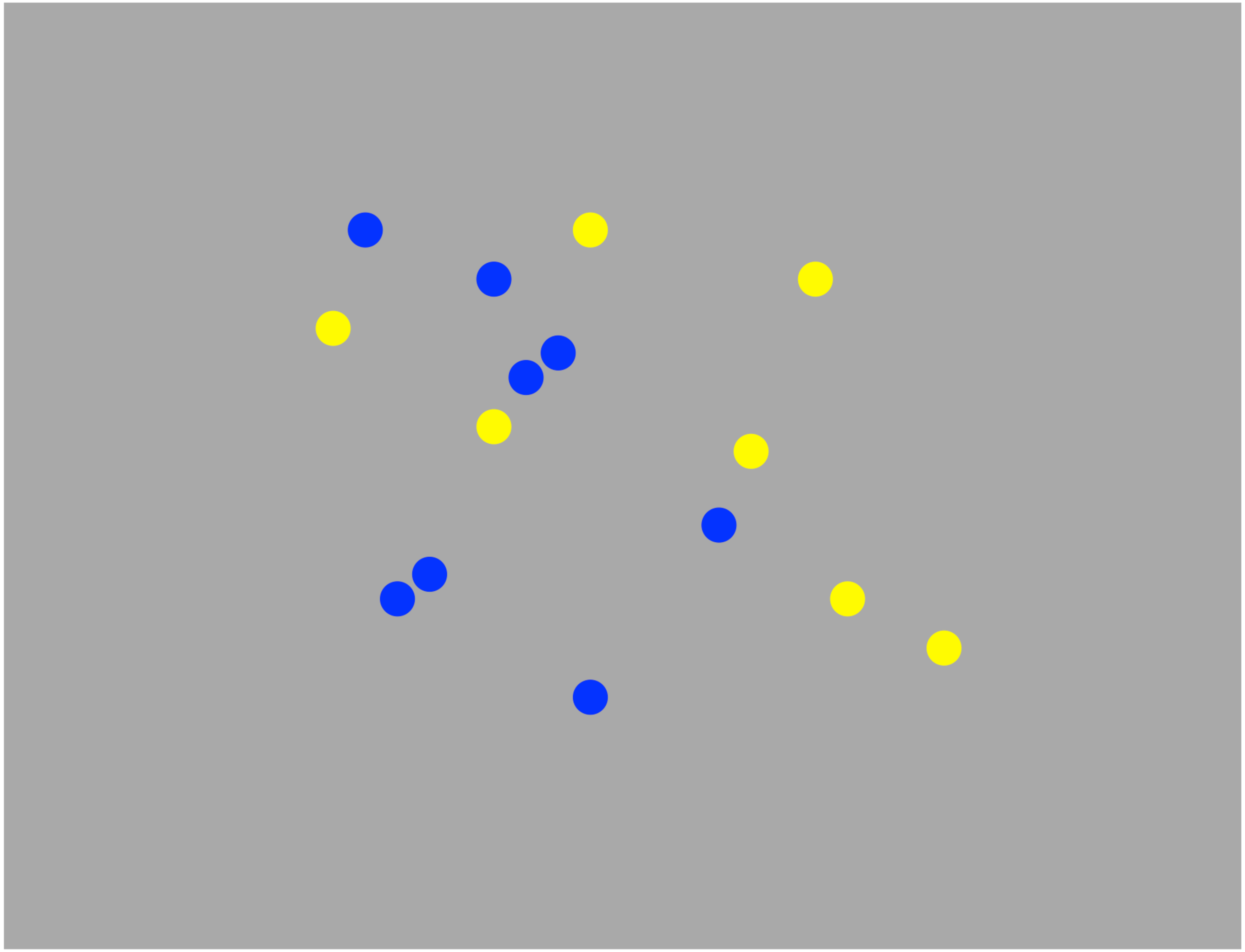
How do we understand language?

- Meaning is a relation between language and the world
- Meaning is a cognitive concept
- Cognitive science provides abundance of experiments

Clark'76, Moxey & Sanford'93, Pinker'07, Berwick & Chomsky'15

MOST OF THE DOTS ARE BLUE

MOST OF THE DOTS ARE BLUE



Hackl'09, Pietroski et al.'09, Geurts et al.'10, Lidz et al.'11, Szymanik et al.'15

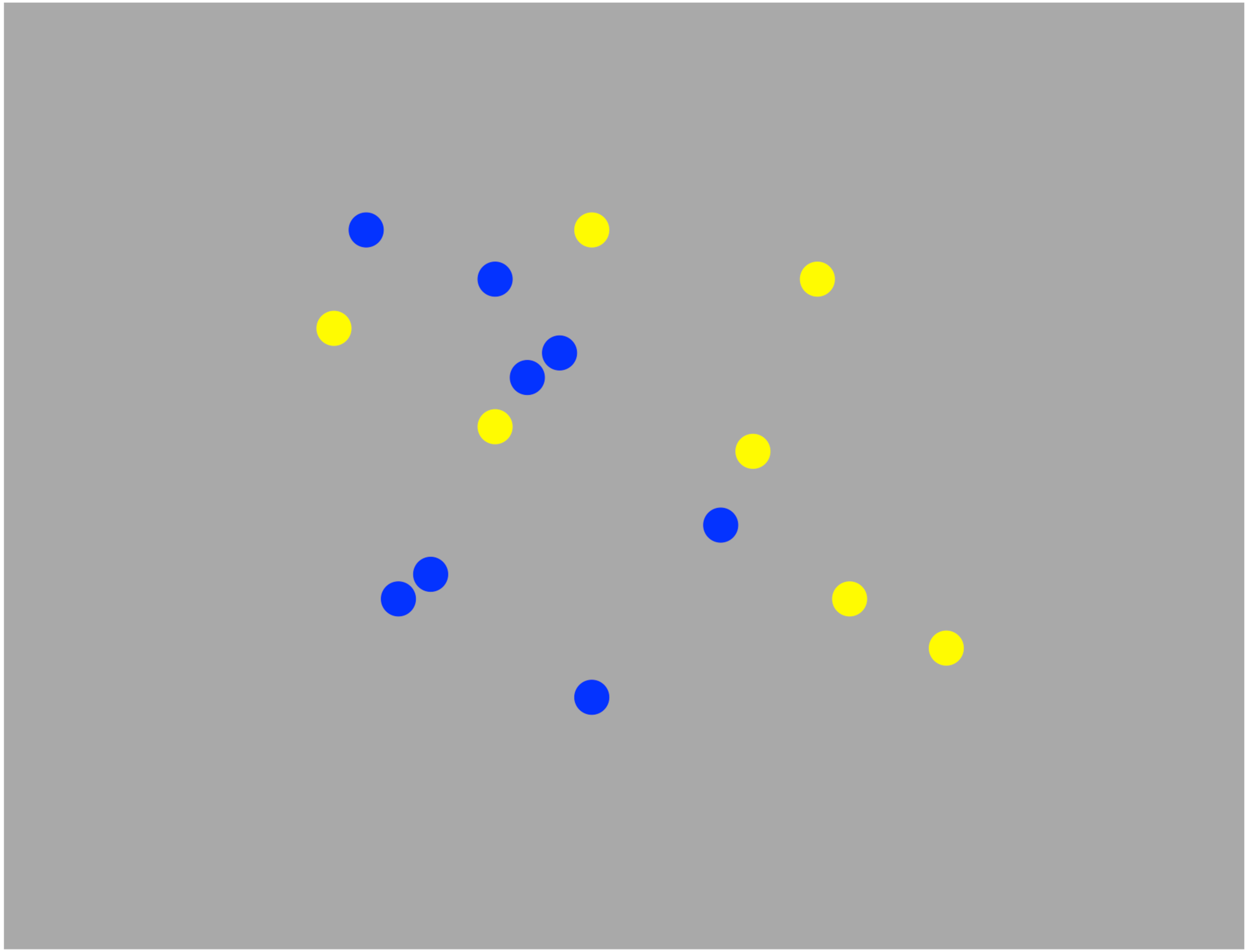
QUANTIFIERS

- Expressions that appear to be descriptions of quantity.
- All, not quite all, nearly all, an awful lot, a lot, a comfortable majority, most, many, more than n, less than n, quite a few, quite a lot, several, not a lot, not, many, only a few, few, a few, hardly any, one, two, three.
- The whole field of study Generalized Quantifier Theory

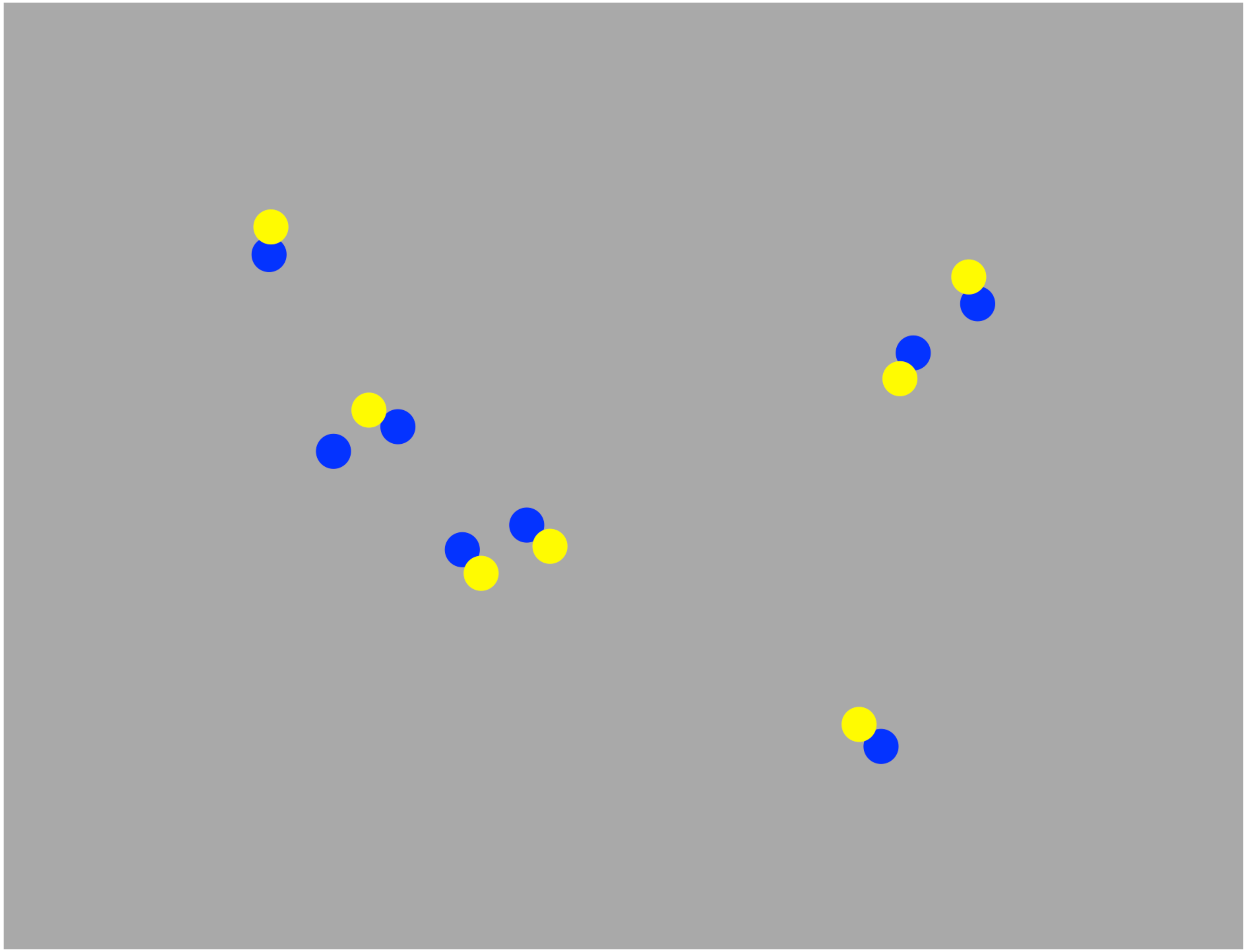
Peters & Westerståhl'08; Szymanik'16

SOME OF THE DOTS ARE BLUE

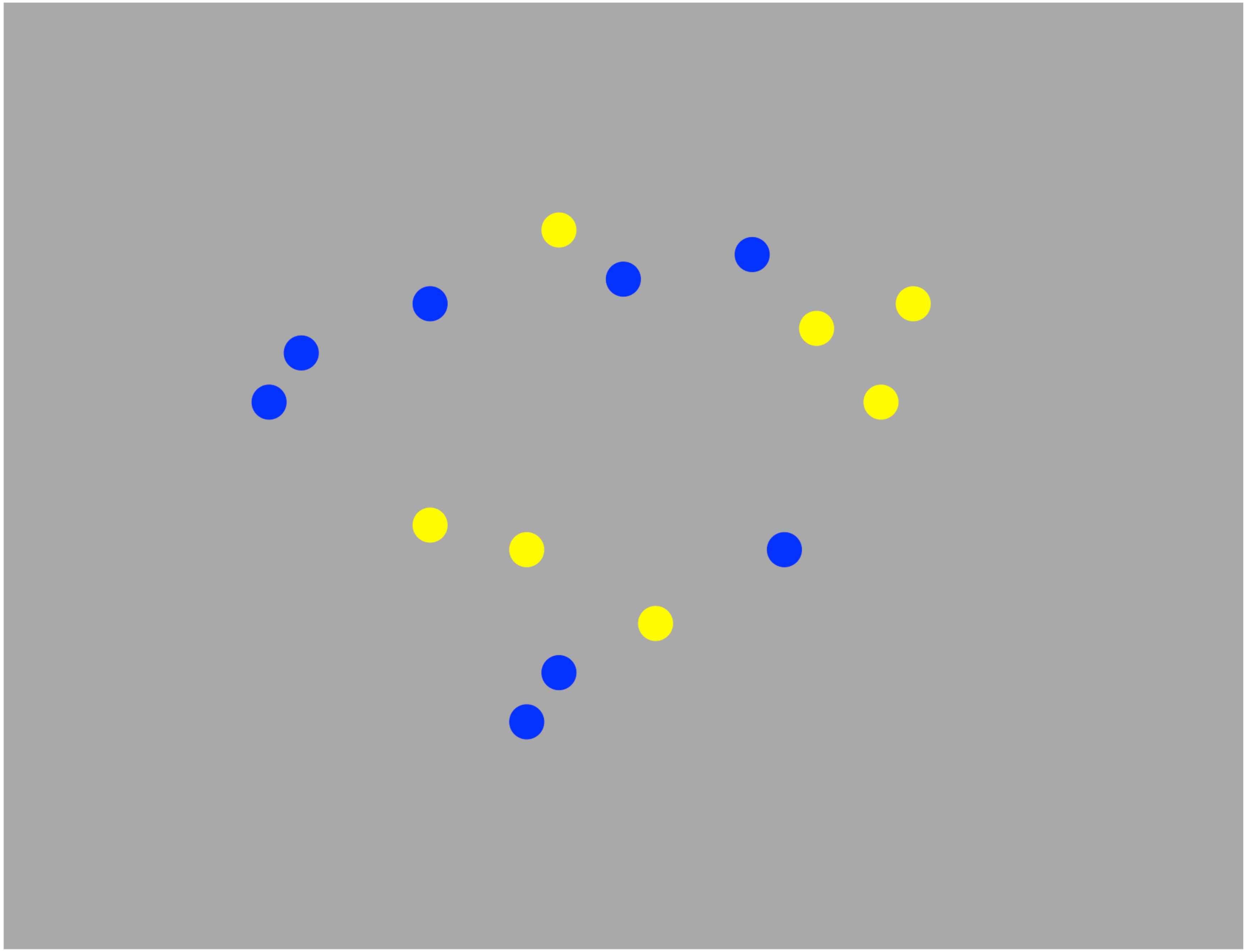
SOME OF THE DOTS ARE BLUE



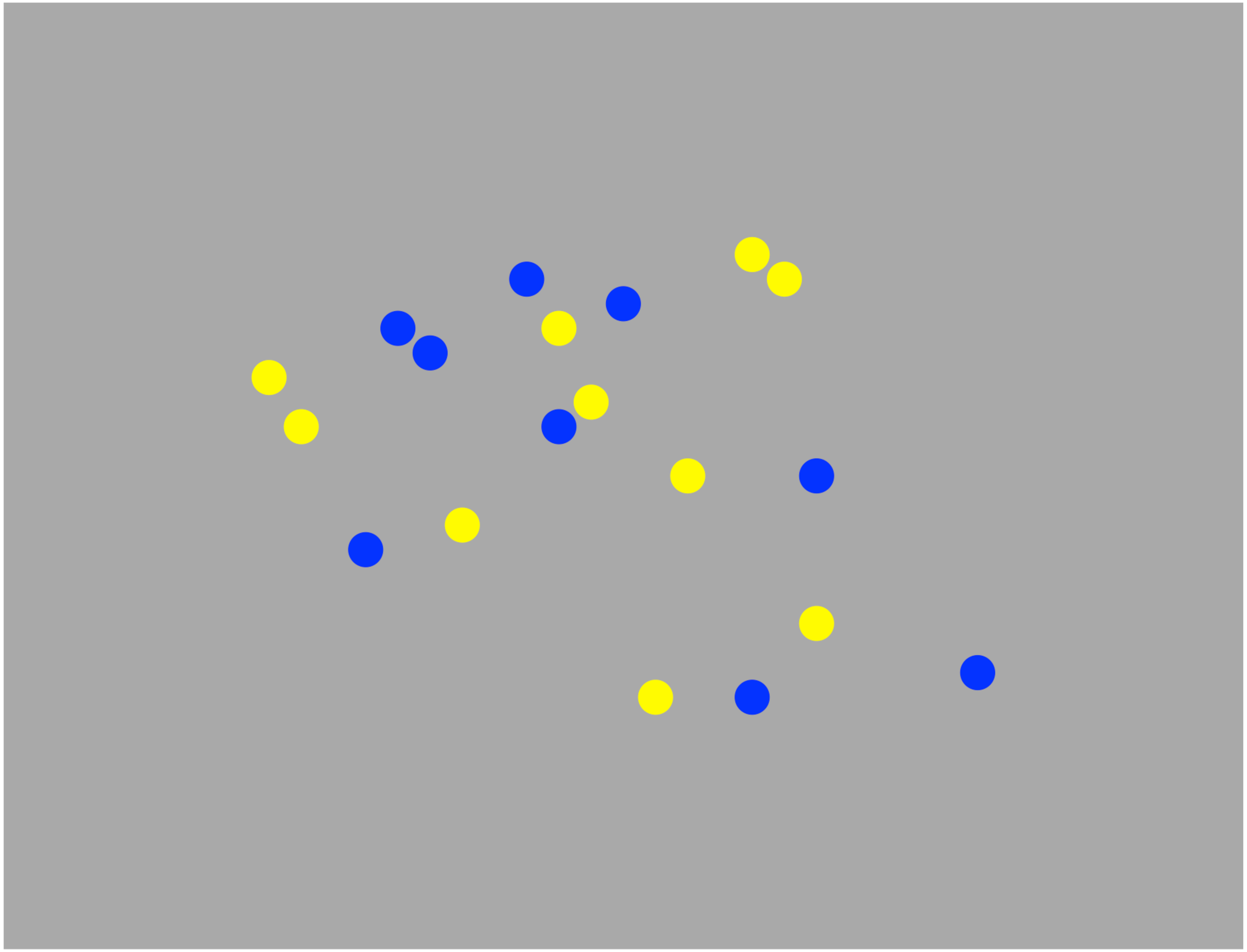
MORE THAN 5 OF THE DOTS ARE BLUE



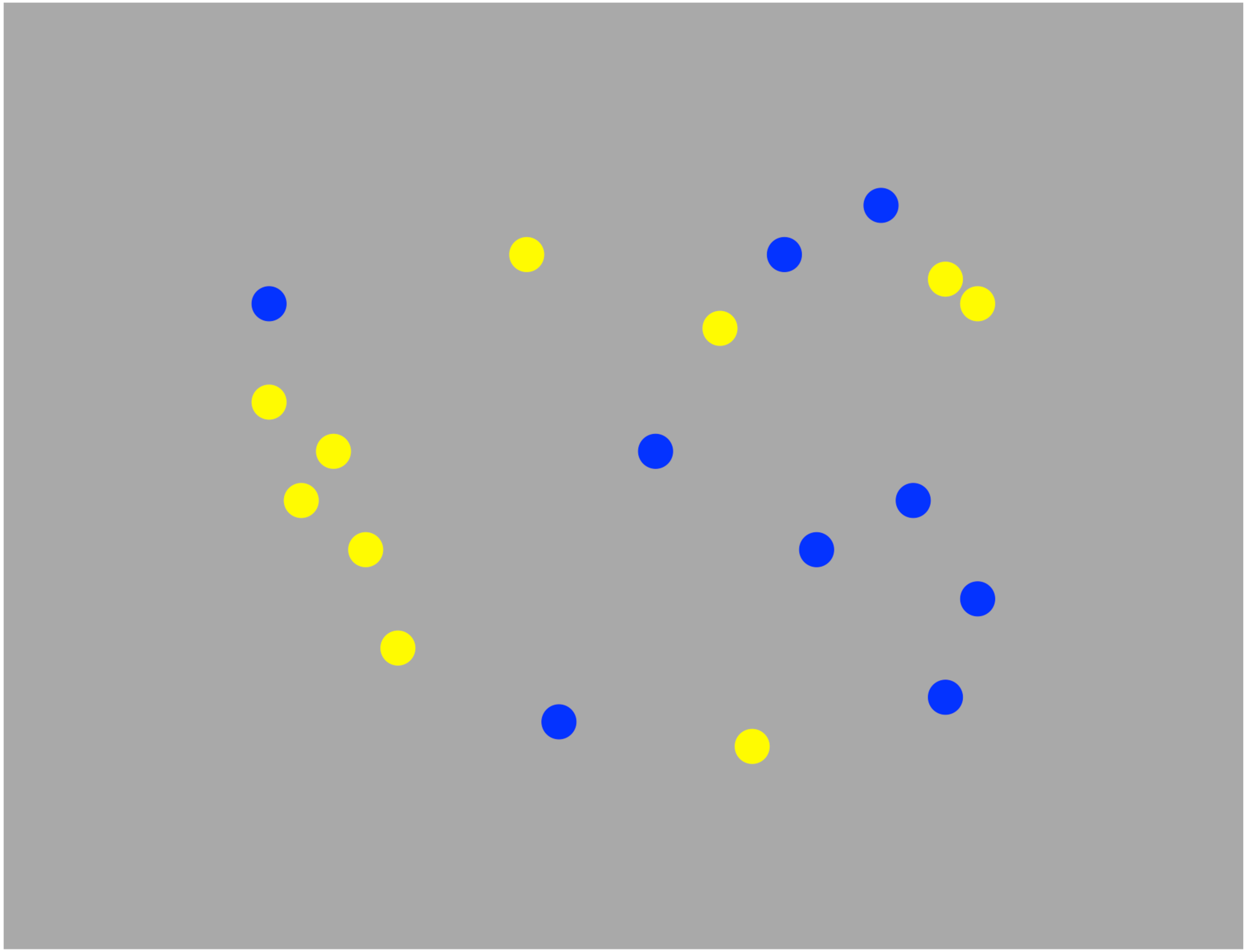
FEWER THAN 7 OF THE DOTS ARE BLUE



AN EVEN NUMBER OF THE DOTS ARE BLUE

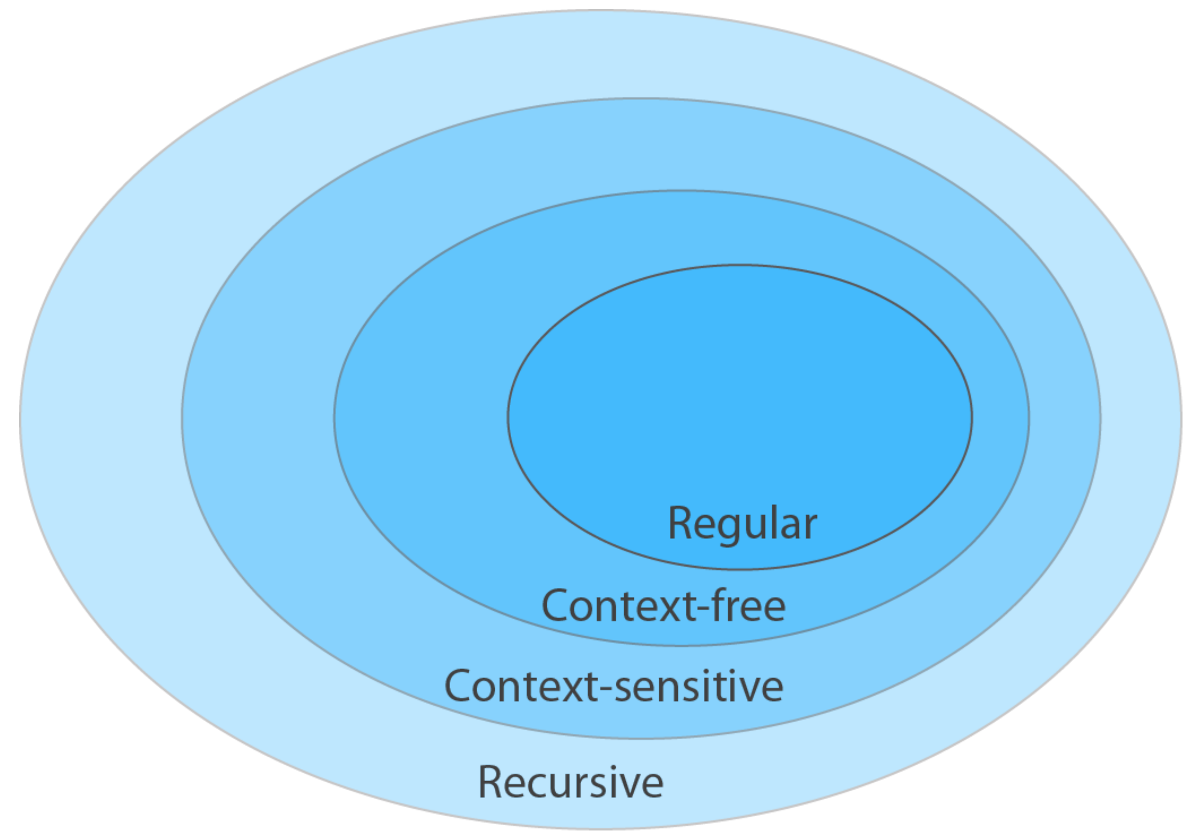


LESS THAN HALF OF THE DOTS ARE BLUE

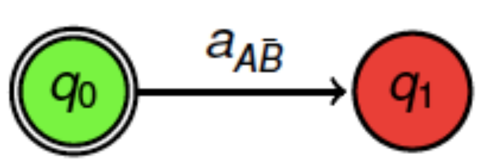


LOGIC & COMPLEXITY CLASSIFICATIONS

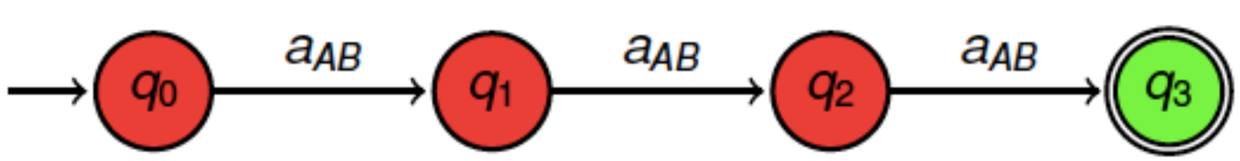
Barwise & Cooper, `81; van Benthem, '86;
Stanley & Westerståhl, '06; Kontinen & Szymanik, '14;
Szymanik, `16



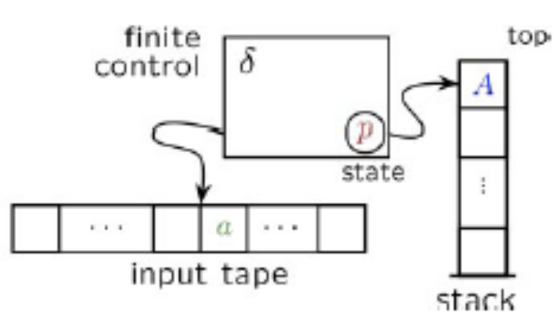
All As are B.

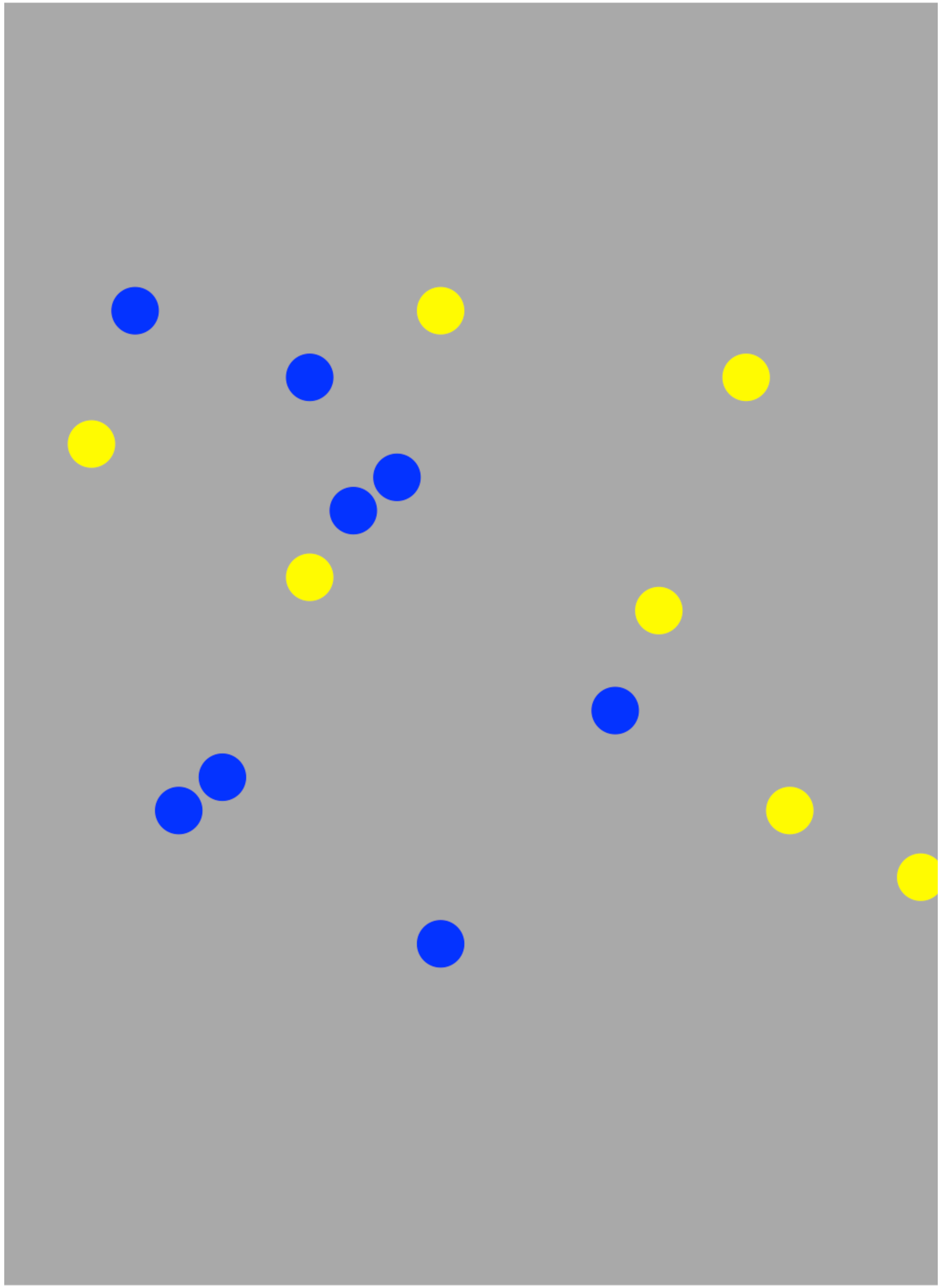


More than 2 As are B.



Most As are B.





DRAW AUTOMATA

- **Some dots are blue.**
- All dots are blue.
- No dots are blue.
- Some dots are not blue.
- **More than 3 dots are blue.**
- Fewer than 4 dots are blue.
- **An even number of dots are blue.**
- An odd number of dots are blue.
- **Most dots are blue.**
- Less than half dots are blue.

CLASSIFYING MINIMAL COMPLEXITY

- Aristotelian quantifiers, e.g, all, some, no, some-not (2-state FA)
- Numerical quantifier, e.g, more than 5 (FA)
- Proportional quantifier, e.g., most (PDA)

van Benthem'86, Mostowski'98,

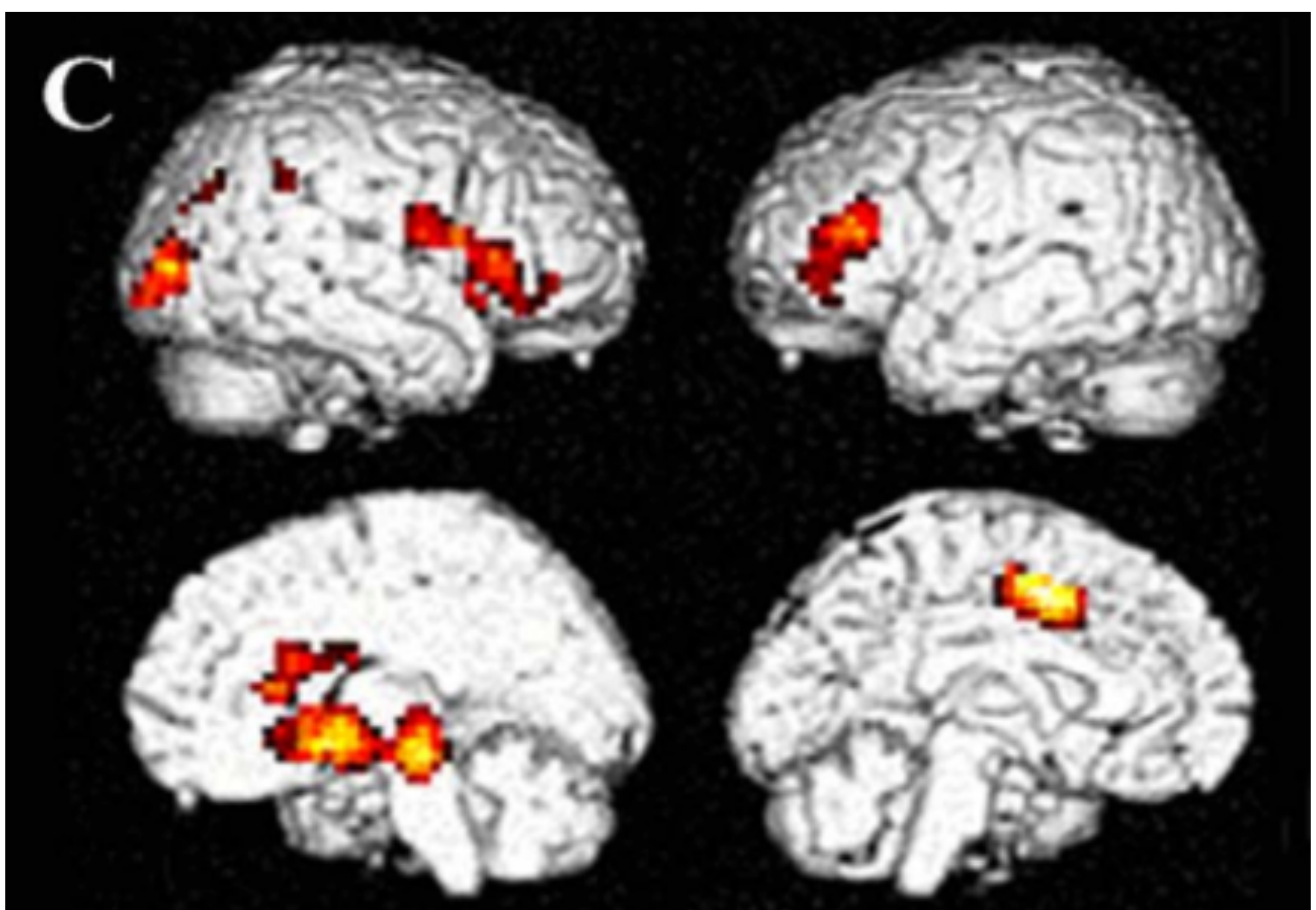
Kanazawa'13; Steinert-Threlkeld & Icard'13, Szymanik'16

ARE LOGICAL DISTINCTIONS PLAUSIBLE?

Differences in brain activity:

A. All quantifiers are associated with numerosity: recruit right inferior parietal cortex.

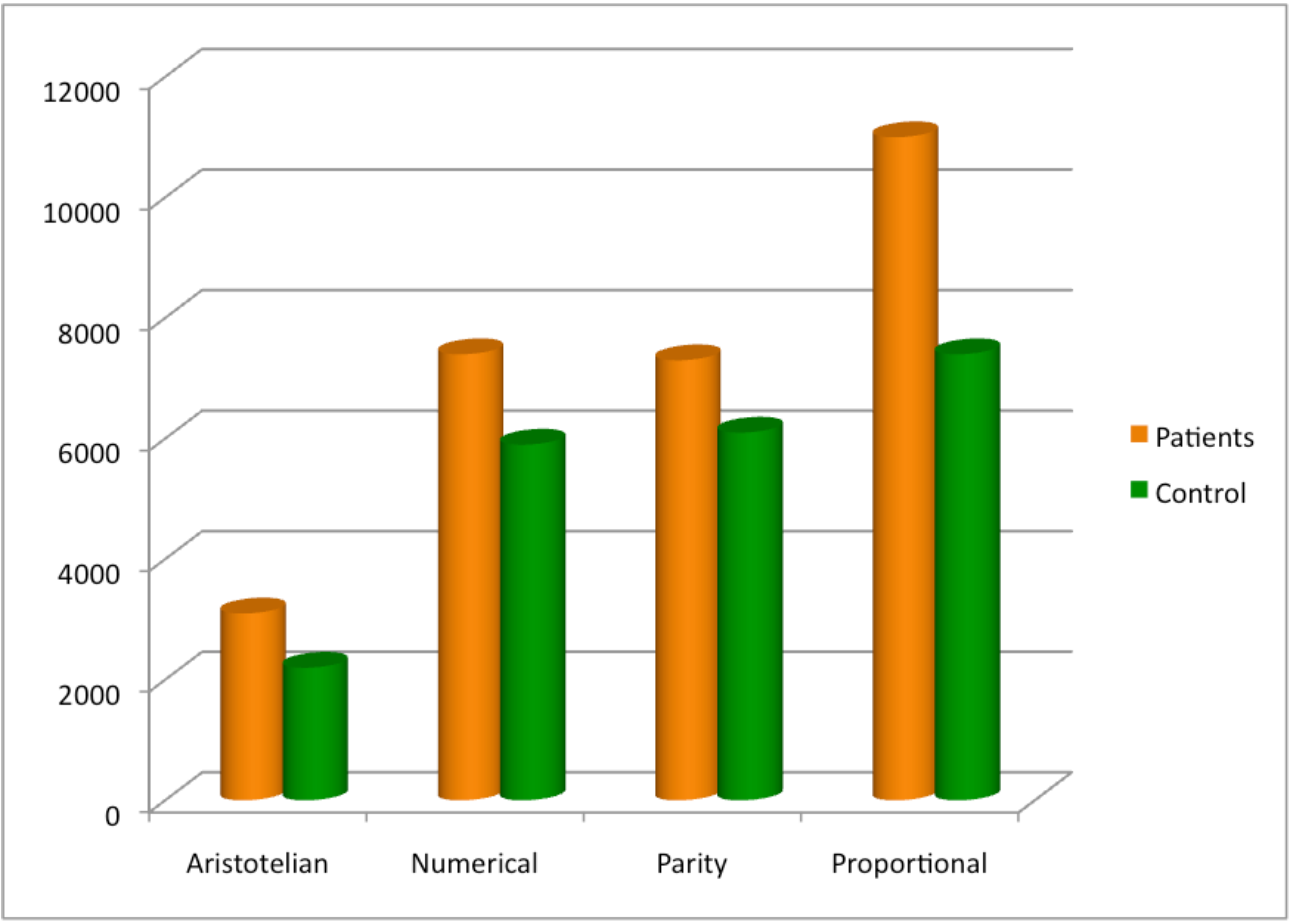
B. Only higher-order activate working-memory capacity: recruit right dorsolateral prefrontal cortex.

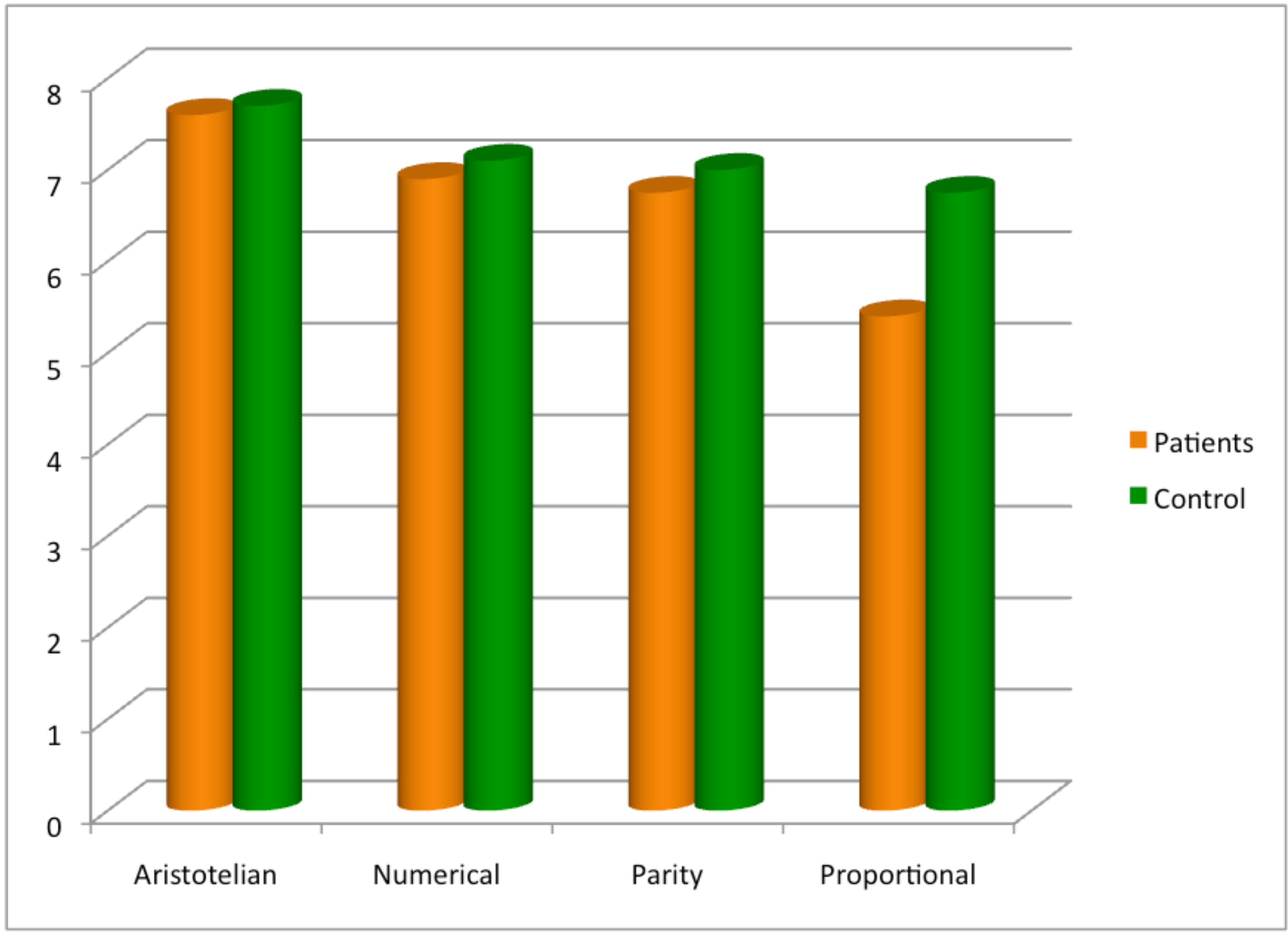


McMillan et al.'05, '06, Szymanik'07

ARE LOGICAL DISTINCTIONS PLAUSIBLE?

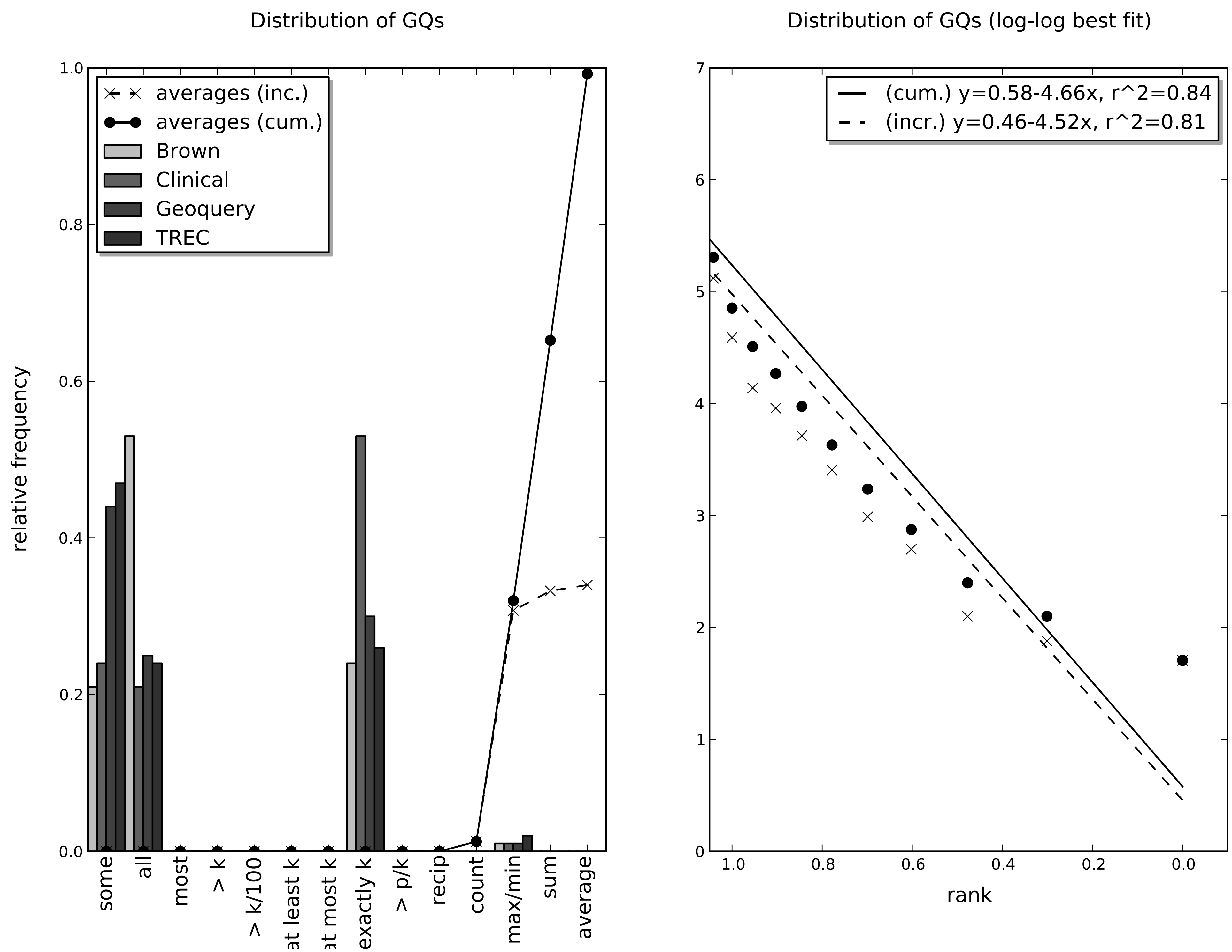
Behavioral differences:





Szymanik & Zajenkowski'10, Zajenkowski et al.'11, Szymanik'16

PRINCIPLE OF MINIMAL EFFORT



Do such measures predict corpora distributions?

Thorne & Szymanik'15

TOPIC 3: PROBLEM SOLVING

Mastermind: an inductive inquiry game, trials of experimentation and evaluation

TOPIC 3: PROBLEM SOLVING

Mastermind: an inductive inquiry game, trials of experimentation and evaluation

Great inductive game to play is Eleusis, see here for the rules, examples and a bit of references.

MASTERMIND: A CODE-BREAKING GAME

- The set consists of:
  - a decoding board
  - code pegs of k colors
  - and feedback pegs of black and white
- Players:
  - the code-maker: chooses a secret pattern of l code pegs
  - the code-breaker: guesses the pattern, in a given n rounds
- Rounds:
  - code-breaker makes a guess by placing a row of l code pegs
  - code-maker provides the feedback:
    - one black for each code peg of correct color and position, and
    - one white for each peg of correct color but wrong position
  - repeat until either the code-breaker guesses correctly, or n incorrect guesses
- Winning:
  - for the code-breaker: if obtains the solution within n rounds
  - the code-maker wins otherwise

PREVIOUS RESEARCH

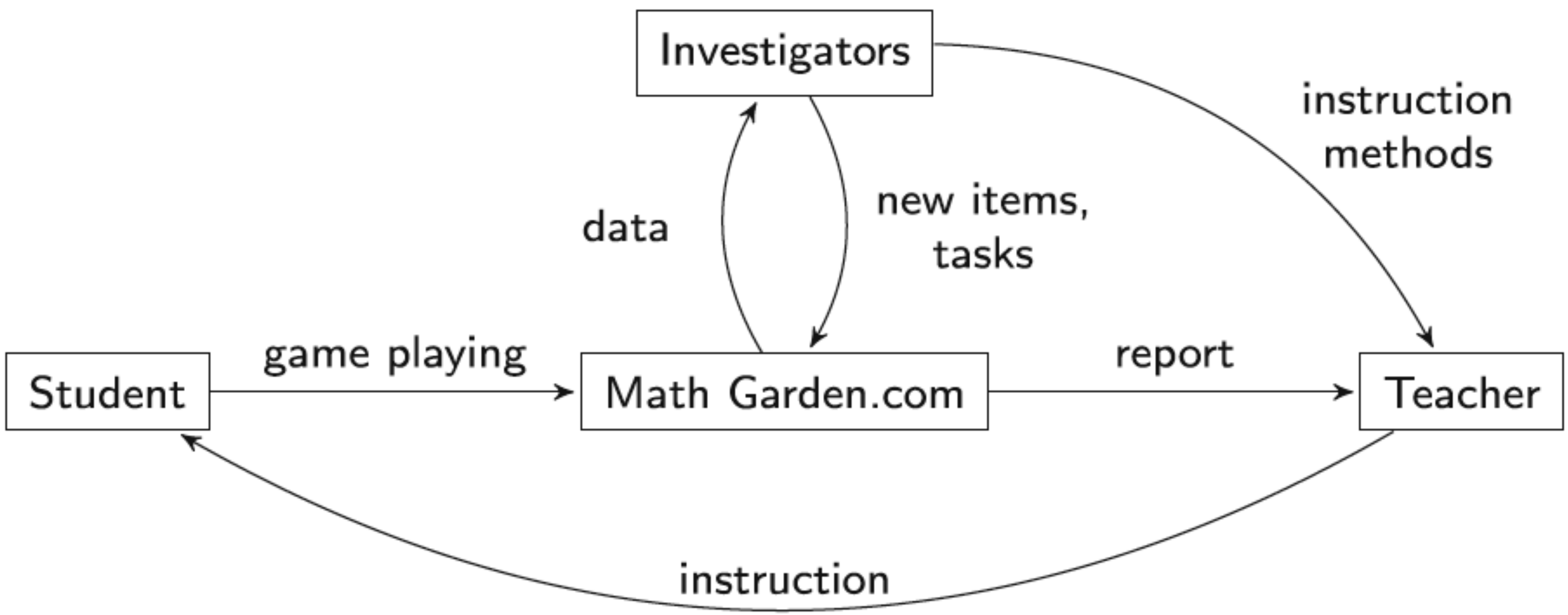
- Acquisition of ToM (Verbrugge & Mol `08)
- Efficient strategies (Knuth ’77, Kooi `05)
- Computational complexity (Stuckman and Zhang ’06)

Mastermind Satisfiability Decision Problem:

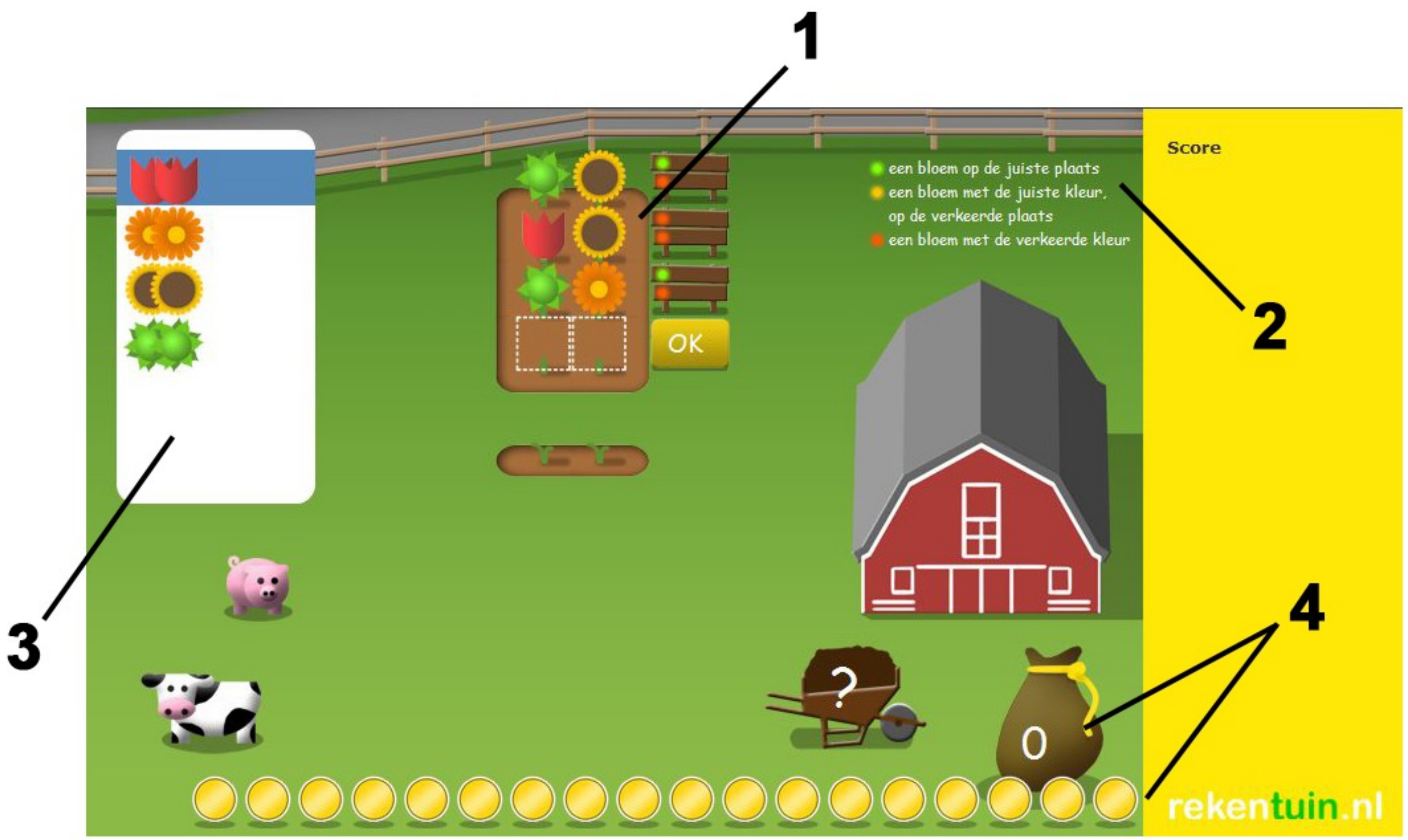
Input: A set of guesses G and their corresponding feedbacks.

Question: Is there at least one valid solution?

MATHGARDEN.COM



DEDUCTIVE MASTERMIND: FLOWERCODE IN MATH GARDEN



Gierasimczuk et al.'13

- decoding board
- short feedback instruction
- domain of flowers to choose from
- timer in the form of disappearing coins

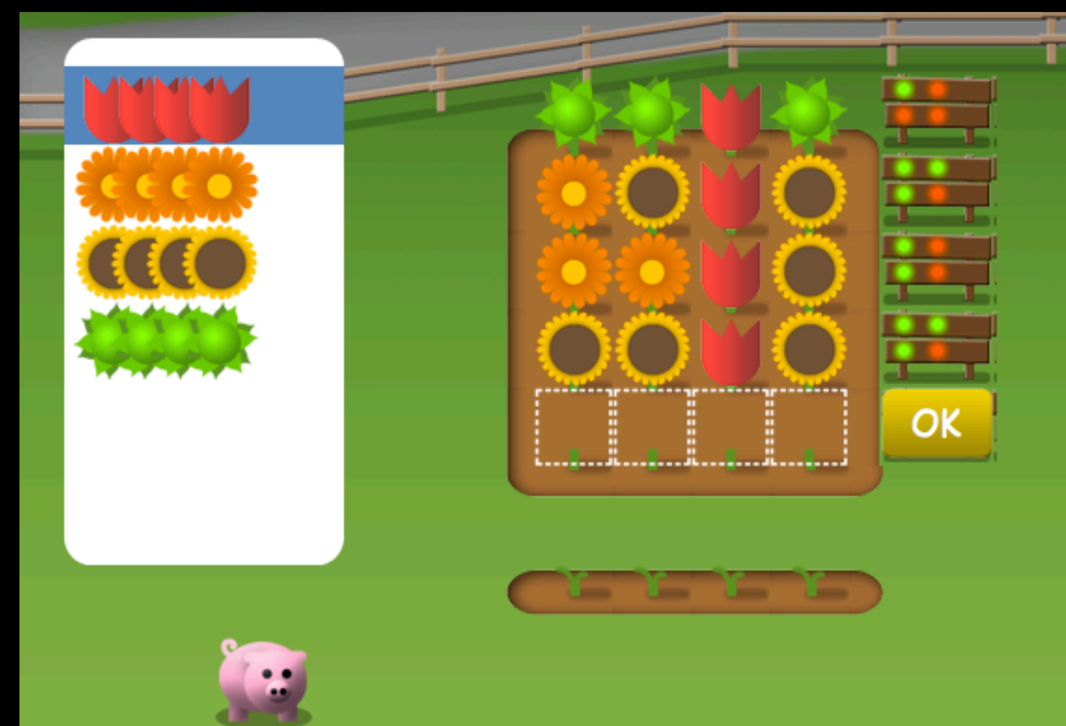
SOME FACTS ABOUT FLOWERCODE

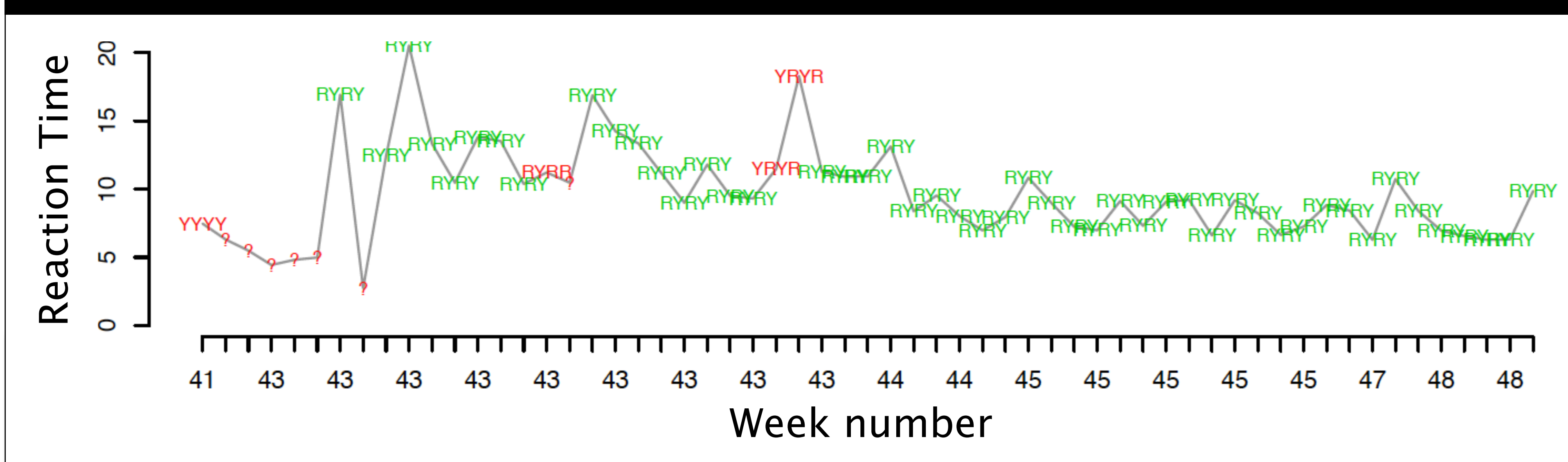
- Atomic logical steps of non-linguistic logical reasoning
- running since November 2010
- 321 game-items, 1-5 flowers, 2-5 colors
- by December 2012, 4,895,648 items had been played
- 37,339 primary school students (grades 1-6, age: 6-12 years)
- in over 700 locations (schools and family homes)

SOME FACTS ABOUT FLOWERCODE

user 163545
played 3601 items

59 times this item





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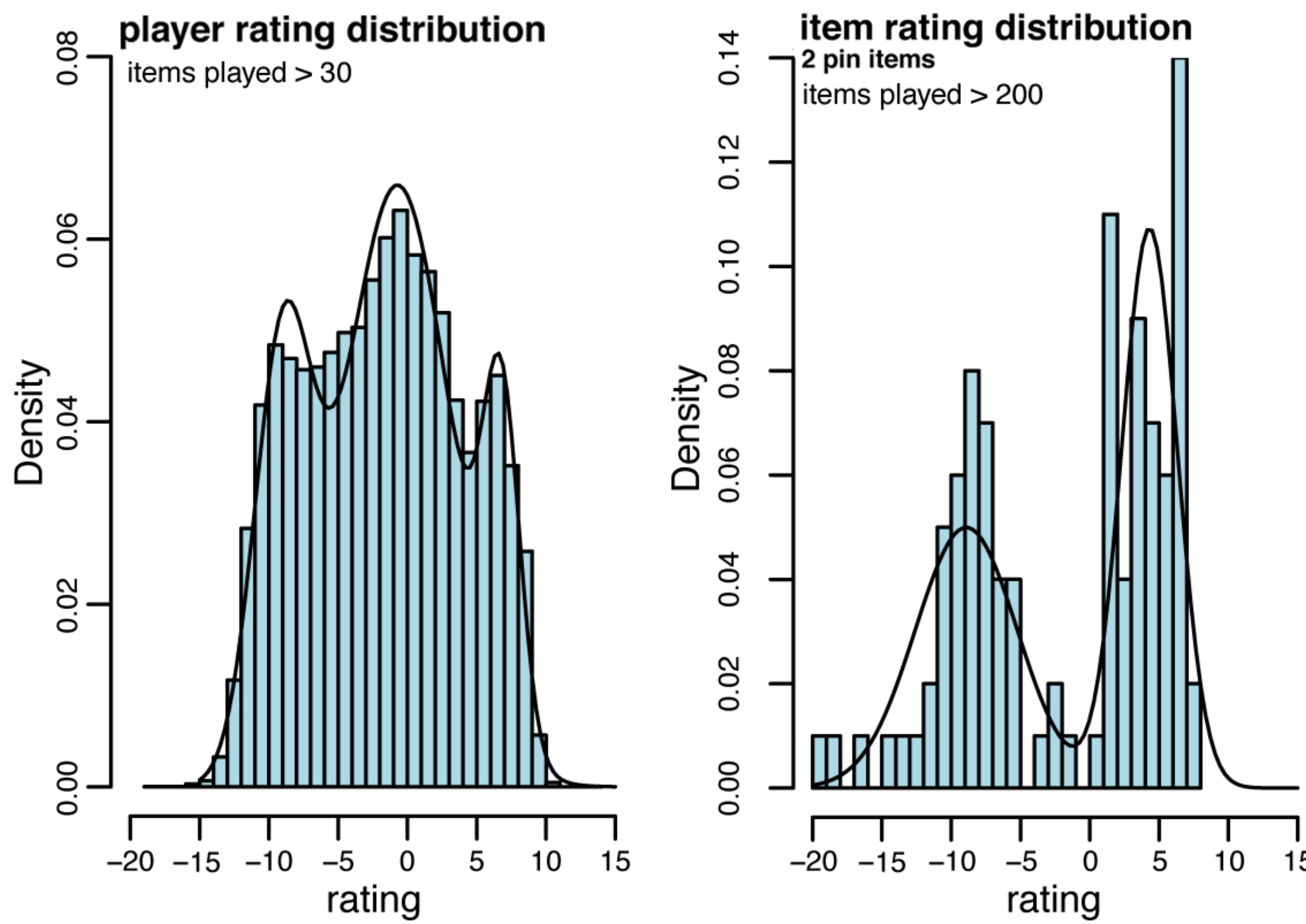
DIFFICULTY LEVELS

- students play game-items suited for their level
- the tasks' difficulty and the students' level are estimated
- via the Elo (1978) rating system
- ratings depend on accuracy and speed of item solving
- By-products:
  1) rating of all items (item difficulty parameters)
  2) rating of children (reflecting the reasoning ability)

NECESSITY OF PRIOR DIFFICULTY ASSESSMENT

initial difficulty estimation in terms of non-logical aspects
(# of flowers, colors, lines, the rate of the hypotheses elimination)



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how to fix this to facilitate the training effect?

A LOGICAL ANALYSIS: CONJECTURES

Each game-item consists of a sequence of conjectures:

Definition

A conjecture of length l over k colors is any sequence given by a total assignment, $h : \{1, \ldots, \ell\} \rightarrow \{c_1, \ldots, c_k\}$. The *goal sequence* is a distinguished conjecture, $goal : \{1, \ldots, \ell\} \rightarrow \{c_1, \ldots, c_k\}$.

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A LOGICAL ANALYSIS: FEEDBACK

- every non-goal conjecture is accompanied by a feedback
- that indicates how similar h is to the given goal assignment
- feedback colors g, o, r

Definition

Let h be a conjecture and let *goal* be the goal sequence, both of length l over k colors. The feedback f for h with respect to *goal* is a sequence

$$\overbrace{g \dots g}^{a} \overbrace{o \dots o}^{b} \overbrace{r \dots r}^{c} = g^a o^b r^c,$$

where $a, b, c \in \mathbb{N}$ and $a + b + c = \ell$.

The feedback consists of:

- exactly one g for each $i \in G$, where $G = \{i \in \{1, \dots \ell\} \mid h(i) = goal(i)\}$.
- exactly one o for every $i \in O$, where $O = \{i \in \{1, \dots, \ell\} \backslash G \mid \exists j \in \{1, \dots, \ell\} \backslash G,$ such that $i \neq j$ and $h(i) = goal(j)\}$.
- exactly one r for every $i \in \{1, \dots, \ell\} \backslash (G \cup O)$.

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THE INFORMATIONAL CONTENT

a second-order formula that encodes any feedback

$g^a o^b r^c$ for any h wrt goal

$$\exists G \subseteq \{1,\ldots\ell\}(card(G)=a \wedge \forall i{\in}G\ h(i)=goal(i) \wedge \forall i{\notin}G\ h(i)\neq goal(i)$$
$$\wedge\ \exists O \subseteq \{1,\ldots\ell\}\backslash G\ (card(O)=b \wedge \forall i{\in}O\ \exists j{\in}\{1,\ldots\ell\}\backslash G(j{\neq}i \wedge h(i)=goal(j))$$
$$\wedge\ \forall i{\in}\{1,\ldots\ell\}\backslash(G{\cup}O)\ \forall j{\in}\{1,\ldots\ell\}\backslash G\ h(i)\neq goal(j))).$$

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THE INFORMATIONAL CONTENT

a general method of providing a propositional formula for any (h, f)

- literals: $h(i) = goal(j)$, where $i, j \in \{1, \dots \ell\}$ (or $p_{i,j}$, for $i, j \in \{1, \dots \ell\}$)
- $\varphi^g_G, \varphi^o_{G,O}, \varphi^r_{G,O}$ correspond to different parts of feedback:
  - $\varphi^g_G := \bigwedge_{i \in G} h(i) = goal(i) \wedge \bigwedge_{j \in \{1,\dots,\ell\} - G} h(j) \neq goal(j)$
  - $\varphi^o_{G,O} := \bigwedge_{i \in O} (\bigvee_{j \in \{1,\dots,\ell\} - G, i \neq j} h(i) = goal(j))$
  - $\varphi^r_{G,O} := \bigwedge_{i \in \{1,\dots\ell\} \setminus (G \cup O), j \in \{1,\dots\ell\} \setminus G, i \neq j} h(i) \neq goal(j)$

as many substitutions of the above as choices of sets G and O

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THE INFORMATIONAL CONTENT

- set $\mathbb{G} := \{G | G \subseteq \{1, \ldots, \ell\} \wedge card(G) = a\}$, and,
- if $G \subseteq \{1, \ldots, \ell\}$, then $\mathbb{O}^G = \{O | O \subseteq \{1, \ldots, \ell\} \backslash G \wedge card(O) = b\}$

Definition

Finally, we can set $Bt(h, f)$, the Boolean translation of (h, f) to be given by:

$$Bt(h, f) := \bigvee_{G \in \mathbb{G}} (\varphi^g_G \wedge \bigvee_{O \in \mathbb{O}^G} (\varphi^o_{G,O} \wedge \varphi^r_{G,O})).$$

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EXAMPLE

Let us take $\ell = 2$ and (h, f) such that: $h(1){:=}c_1$, $h(2){:=}c_2$; $f{:=}or$. Then $\mathbb{G}{=}\{\emptyset\}$, $\mathbb{O}^{\{\emptyset\}}{=}\{\{1\},\{2\}\}$. The corresponding formula, $Bt(h, f)$, is:

$$(goal(1){\neq}c_1 \wedge goal(2){\neq}c_2) \wedge ((goal(1){=}c_2 \wedge goal(2){\neq}c_1) \vee (goal(2){=}c_1 \wedge goal(1){\neq}c_2))$$

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GAME ITEM

Definition

A Deductive Mastermind game-item over ℓ positions, k colors and n lines, $DM(l, k, n)$, is a set $\{(h_1, f_1), \ldots, (h_n, f_n)\}$ of pairs, each consisting of a single conjecture together with its corresponding feedback. Respectively, $Bt(DM(l, k, n)) = Bt(\{(h_1, f_1), \ldots, (h_n, f_n)\}) = \{Bt(h_1, f_1), \ldots, Bt(h_n, f_n)\}$.

- hence, each DM game-item is a set of Boolean formulae
- moreover, by the construction this set is satisfiable
- and, even more, there is a unique valuation

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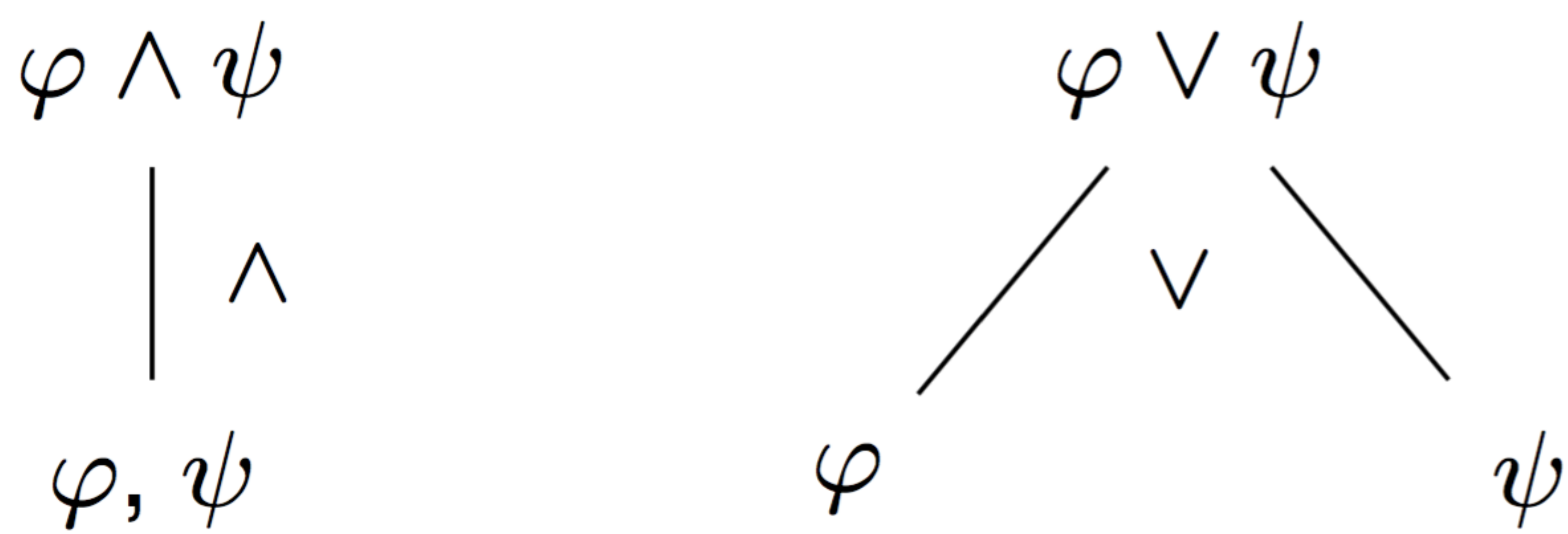
ANALYTIC TABLEAUX FOR DEDUCTIVE

- analytic tableau is a decision procedure for propositional logic
- it solves satisfiability of finite sets of formulas of propositional logic
- by giving an adequate valuation
- building a formula-labeled tree rooted at the set
- unfolding breaks them into smaller formulae
- until contradiction is found or no further reduction is possible



ANALYTIC TABLEAU AND DM

Applying the analytic tableaux method to the Boolean translation of a Deductive Mastermind game-item will give the unique missing assignment **goal**.

2-PLACED GAME-ITEMS

gg, go, oo, rr, gr, or

2-PLACED GAME-ITEMS

gg, go, oo, rr, gr, or

c_i, c_j

$$\Big| \; oo$$

$goal(1) \neq c_i$
$goal(2) \neq c_j$
$goal(1) = c_j$
$goal(2) = c_i$

2-PLACED GAME-ITEMS

gg, go, oo, rr, gr, or

$$c_i, c_j \quad \Big| \; oo$$

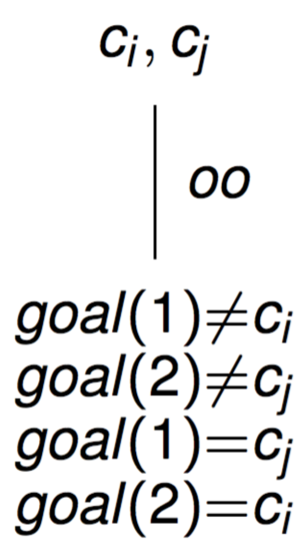
$goal(1) \neq c_i$
$goal(2) \neq c_j$
$goal(1) = c_j$
$goal(2) = c_i$

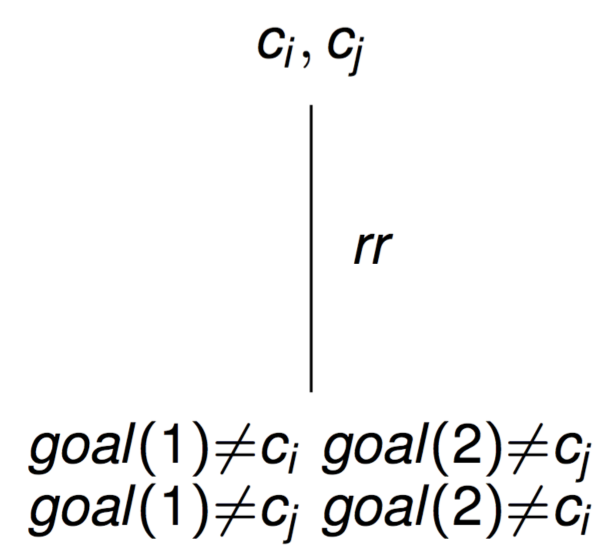
$$c_i, c_j \quad \Big| \; rr$$

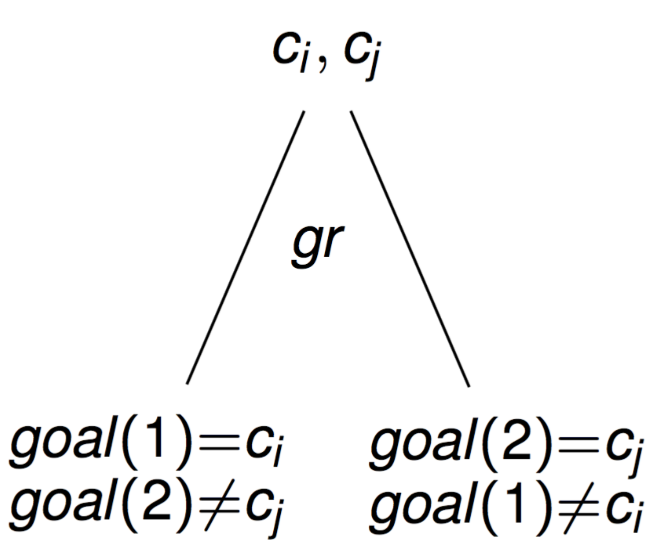
$goal(1) \neq c_i$ $goal(2) \neq c_j$
$goal(1) \neq c_j$ $goal(2) \neq c_i$

2-PLACED GAME-ITEMS

gg, go, oo, rr, gr, or







2-PLACED GAME-ITEMS

gg, go, oo, rr, gr, or

c_i, c_j

oo

$goal(1) \neq c_i$
$goal(2) \neq c_j$
$goal(1) = c_j$
$goal(2) = c_i$

c_i, c_j

rr

$goal(1) \neq c_i$ $goal(2) \neq c_j$
$goal(1) \neq c_j$ $goal(2) \neq c_i$

c_i, c_j

gr

$goal(1) = c_i$
$goal(2) \neq c_j$

$goal(2) = c_j$
$goal(1) \neq c_i$

c_i, c_j

or

$goal(1) \neq c_i$
$goal(2) = c_i$
$goal(1) \neq c_j$
$goal(2) \neq c_j$

$goal(2) \neq c_j$
$goal(1) = c_j$
$goal(2) \neq c_i$
$goal(2) \neq c_i$

2-PLACED GAME-ITEMS

gg, go, oo, rr, gr, or

c_i, c_j

oo

$goal(1) \neq c_i$
$goal(2) \neq c_j$
$goal(1) = c_j$
$goal(2) = c_i$

c_i, c_j

rr

$goal(1) \neq c_i$ $goal(2) \neq c_j$
$goal(1) \neq c_j$ $goal(2) \neq c_i$

c_i, c_j

gr

$goal(1) = c_i$	$goal(2) = c_j$
$goal(2) \neq c_j$	$goal(1) \neq c_i$

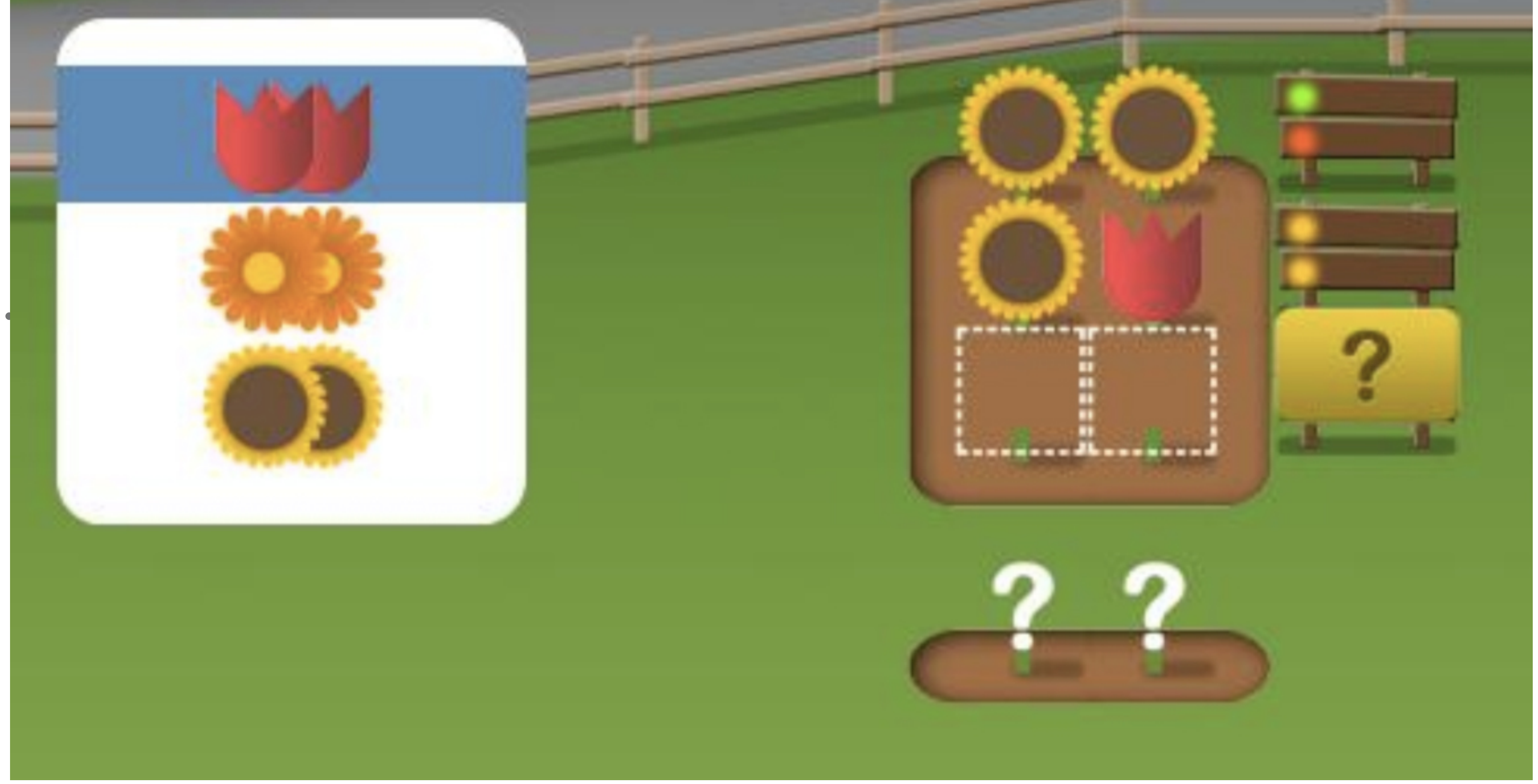
c_i, c_j

or

$goal(1) \neq c_i$	$goal(2) \neq c_j$
$goal(2) = c_i$	$goal(1) = c_j$
$goal(1) \neq c_j$	$goal(2) \neq c_i$
$goal(2) \neq c_j$	$goal(2) \neq c_i$

oo < rr < gr < or

EXAMPLE



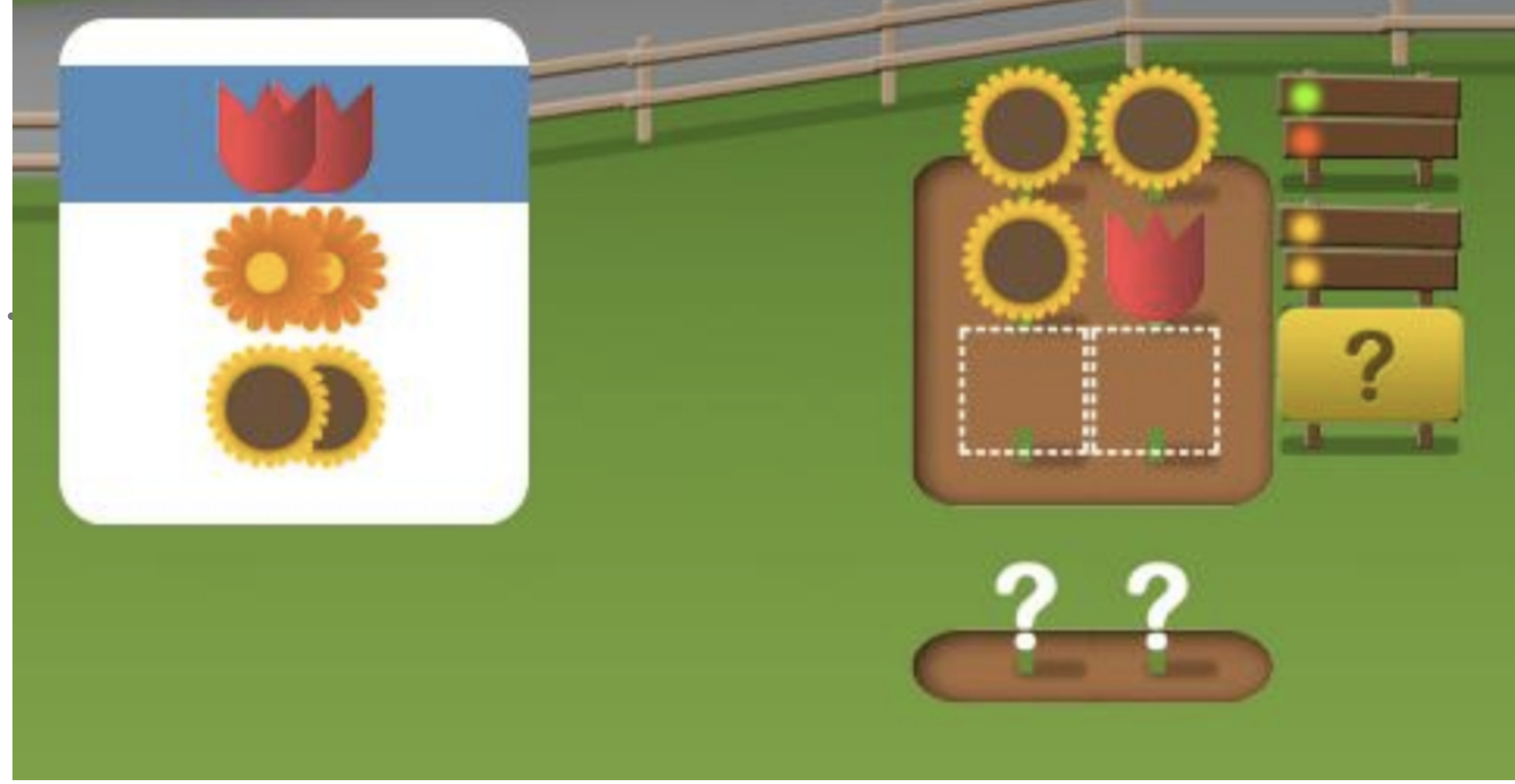
Gierasimczuk et al.'13

EXAMPLE

$$Bt(h_1, f_1)$$
$$Bt(h_2, f_2)$$

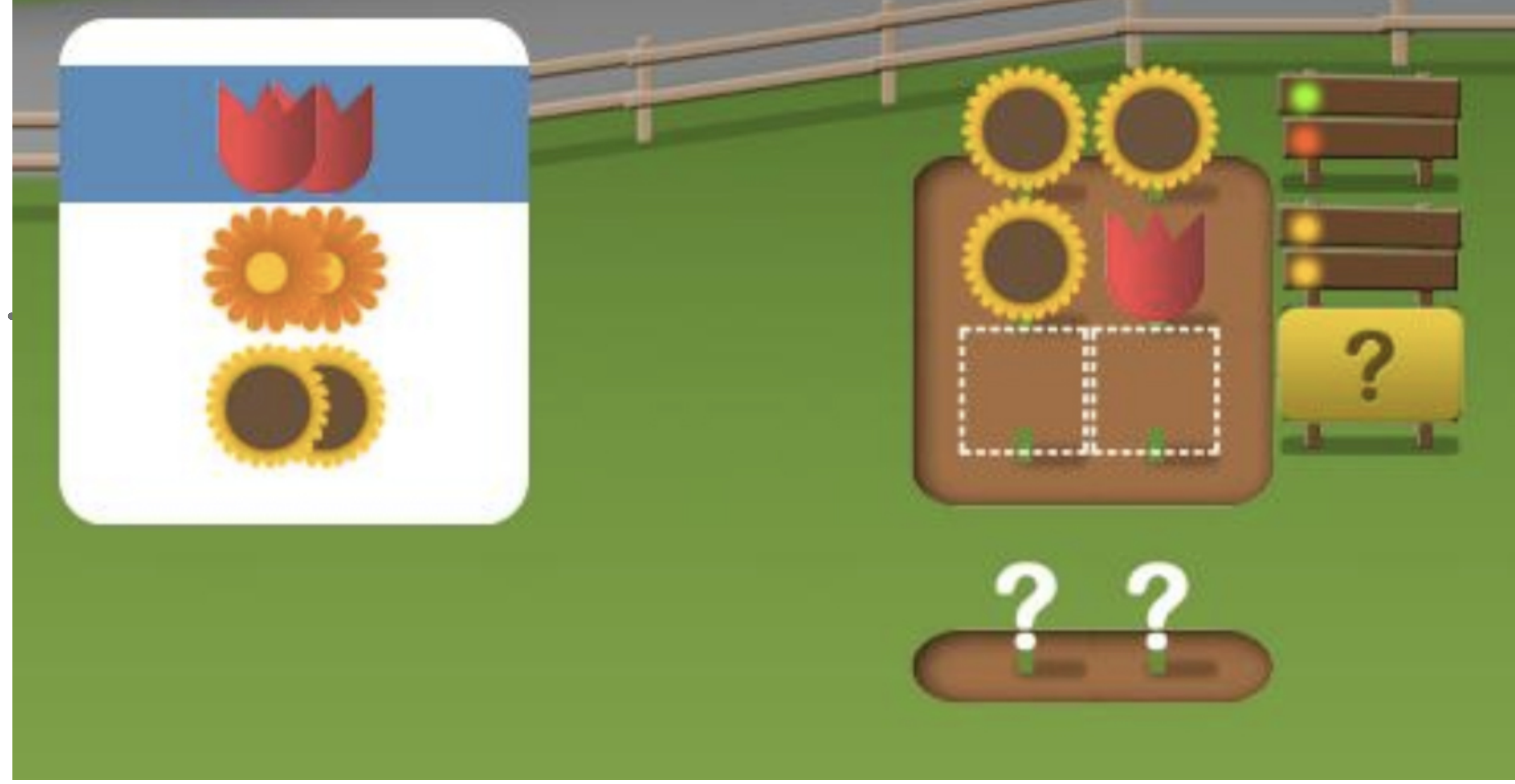
gr

$goal(1) = c_1$	$goal(2) = c_1$
$goal(2) \neq c_1$	$goal(1) \neq c_1$
$Bt(h_2, f_2)$	$Bt(h_2, f_2)$
oo	oo
$goal(1) = c_2$	$goal(1) = c_2$
$goal(2) = c_1$	$goal(2) = c_1$



Gierasimczuk et al.'13

EXAMPLE



$$Bt(h_1, f_1)$$
$$Bt(h_2, f_2)$$

$$gr$$

$$\begin{array}{ll} goal(1)=c_1 & goal(2)=c_1 \\ goal(2)\neq c_1 & goal(1)\neq c_1 \\ Bt(h_2, f_2) & Bt(h_2, f_2) \end{array}$$

$$\begin{array}{ll} oo & oo \end{array}$$

$$\begin{array}{ll} goal(1)=c_2 & goal(1)=c_2 \\ goal(2)=c_1 & goal(2)=c_1 \end{array}$$

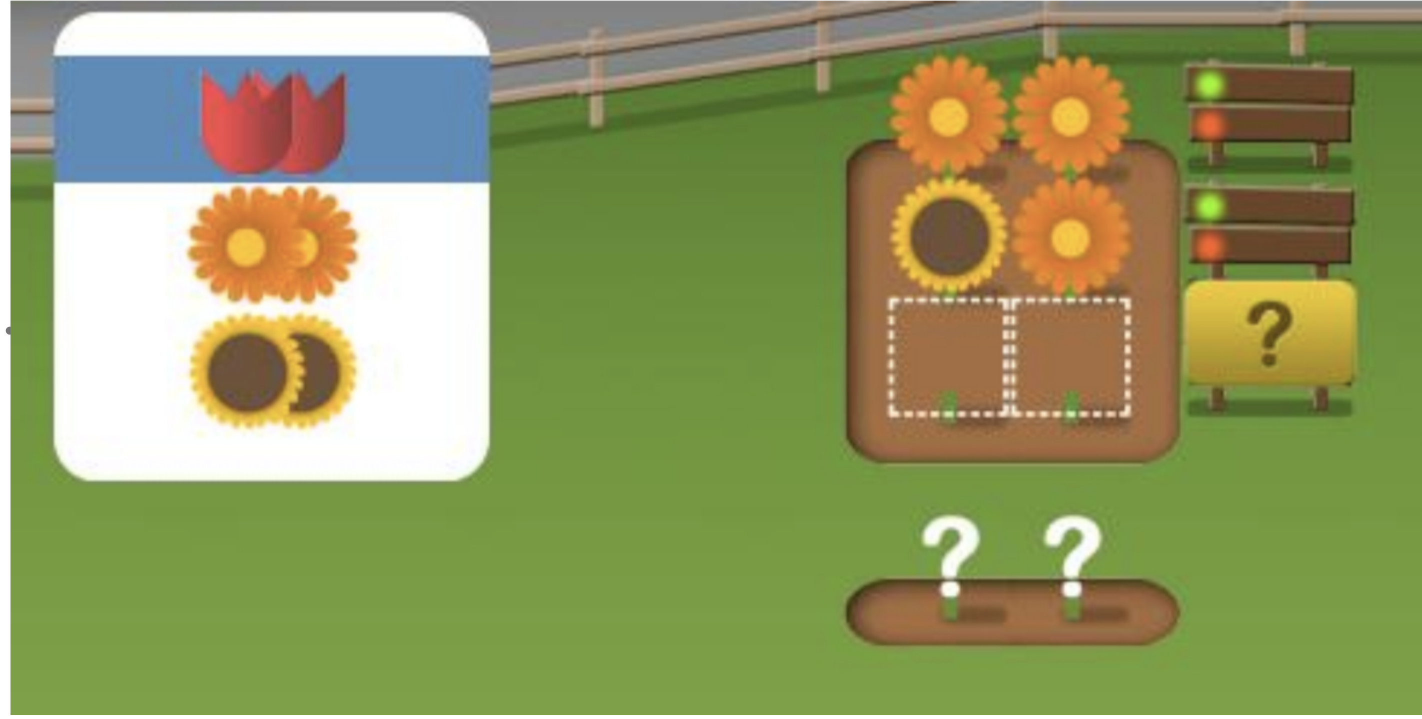
$$Bt(h_2, f_2)$$
$$Bt(h_1, f_1)$$

$$oo$$

$$goal(1)=c_2$$
$$goal(2)=c_1$$
$$Bt(h_1, f_1)$$

Gierasimczuk et al.'13

ANOTHER EXAMPLE



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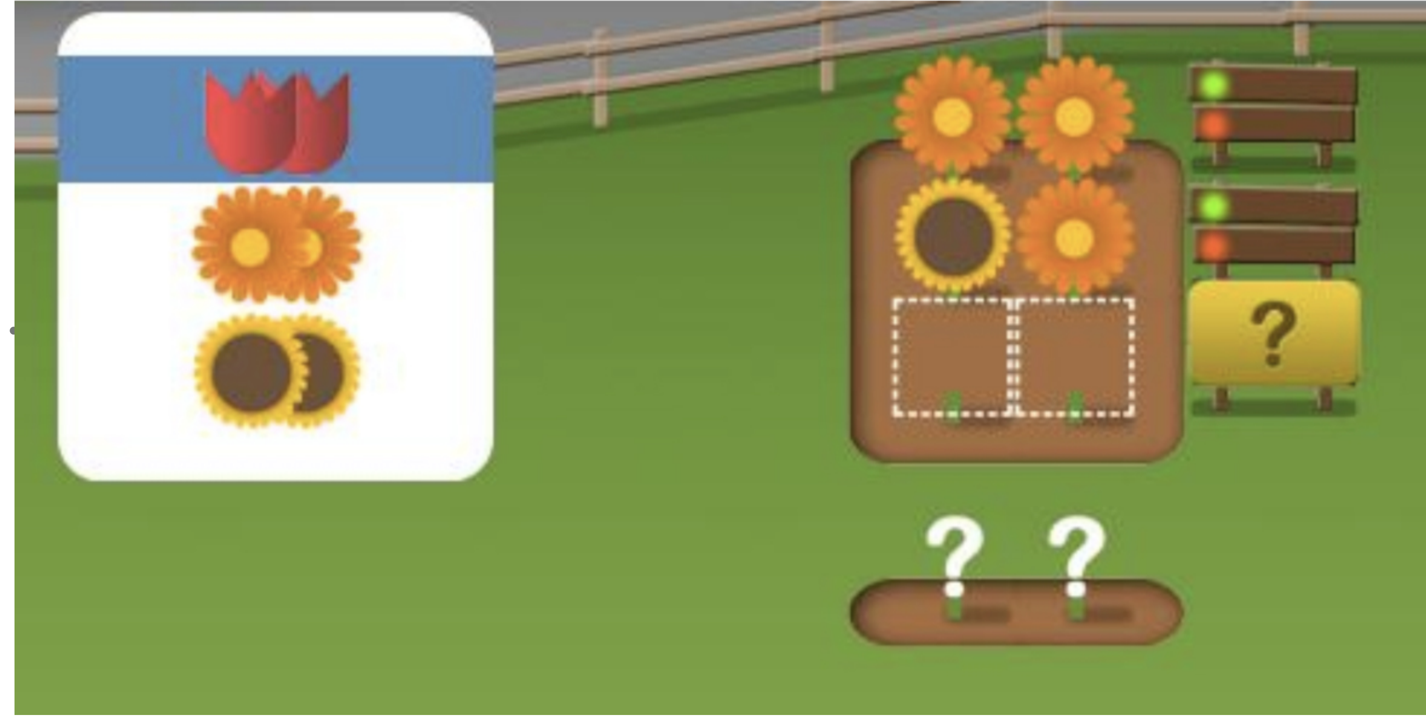
ANOTHER EXAMPLE



Gierasimczuk et al.'13

$$Bt(h_1, f_1) \quad Bt(h_2, f_2)$$

gr

$$goal(1)=c_1 \quad goal(2)\neq c_1 \quad Bt(h_2, f_2) \qquad\qquad goal(2)=c_1 \quad goal(1)\neq c_1 \quad Bt(h_2, f_2)$$

gr $\qquad$ gr

$$goal(1)=c_2 \quad goal(2)\neq c_1 \qquad goal(2)=c_1 \quad goal(1)\neq c_2 \qquad goal(1)=c_2 \quad goal(2)\neq c_1 \qquad goal(2)=c_1 \quad goal(1)\neq c_2$$

HYPOTHESIS AND PRELIMINARY RESULTS

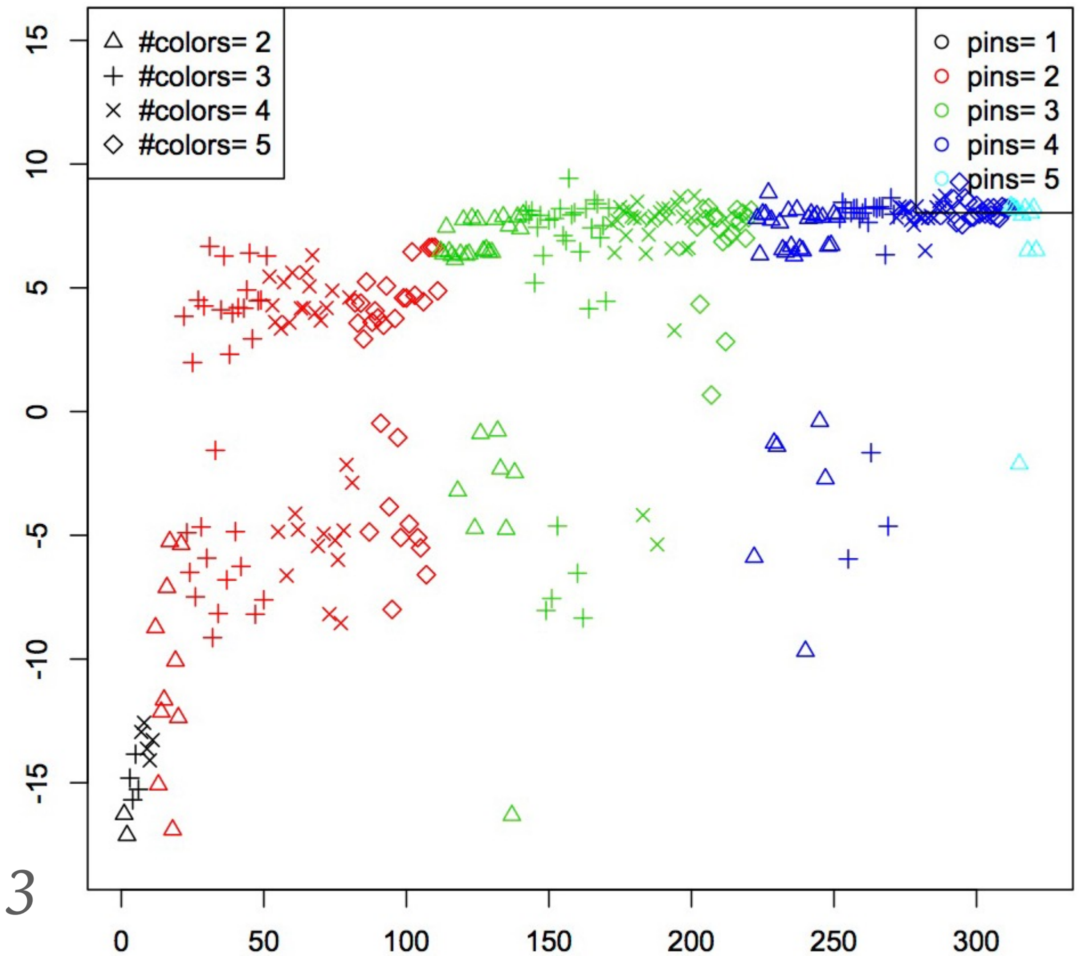
- tableau give ‘ideal’ reasoning scheme
- abstract complexity measure (tree size)
- shape and size of the tree depends on what goes first (minimal)
- reasoning optimization:
  - items’ initial difficulty corresponds to the size of top-botom trees
  - items’ logical difficulty corresponds to the size of the minimal trees
  - the reasoning is optimized according to feedback complexity

METHOD

- participants: 28,247 students from grades 1-6, of age: 6-12 years
- played: 2,187,354 items between Nov. 2010 and Jan. 2012
- items: 321 DM items among them 100 two-places items

RESULTS

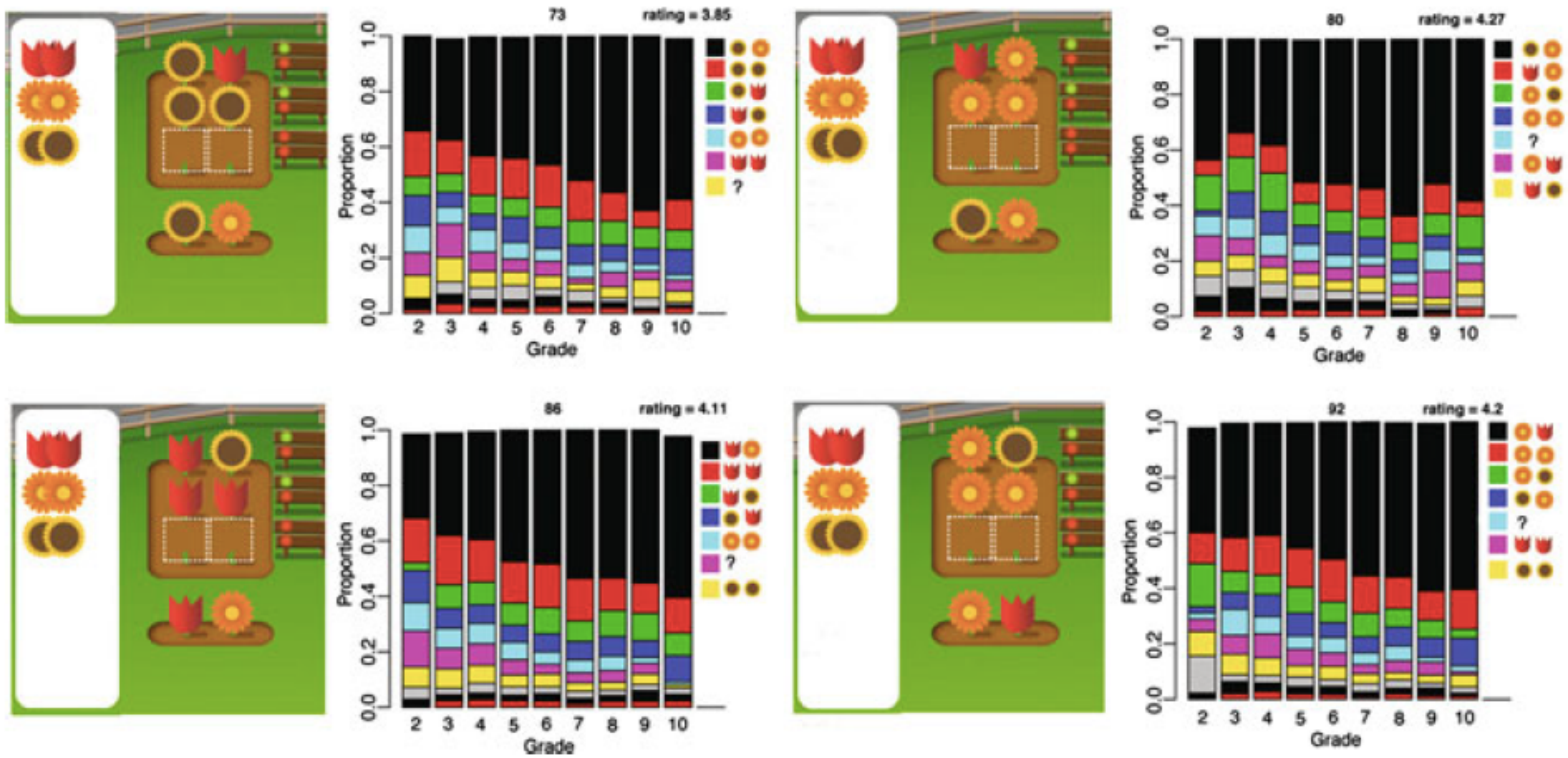
- all factors but one (gr) were significant in predicting item difficulties
- two difficulty clusters:
- easy:
  no or feedback and no gr feedback
  no or feedback, at least one gr feedback, and all colors are included
- difficult: otherwise



Gierasimczuk et al.'13

TOWARD THE ERROR ANALYSIS

- Frequencies of answers are consistent with the analysis.
- Most common erroneous responses are structurally the same within the items that have the same tableau representation



Gierasimczuk et al.'13

Fig. 10 The figure displays the frequencies of various answers to four similar DMM-items. All items have the same logical structure (the same reasoning trees) and display very similar error patterns, see items 73, 86, and 92. In terms of structure, item 80 (*top-right corner*) is a mirror image of the remaining ones. Although similar, it clearly displays a different error pattern. This can be explained via the respective tableaux. The tableau for item 80 requires more steps and thus a higher memory load

SUMMARY

- Non-linguistic task with proof-theoretic analysis
- Complexity of the proof correlates with difficulty.
- Errors seem to follow the `logical’ pattern.
- Various complexity measures:
  - initial item difficulty ~ abstract size of the proof
  - logical item difficulty ~ size of the minimal proof
  - reasoning difficulty ~ optimized algorithm

LEARNING, TEACHING, & STRATEGIES

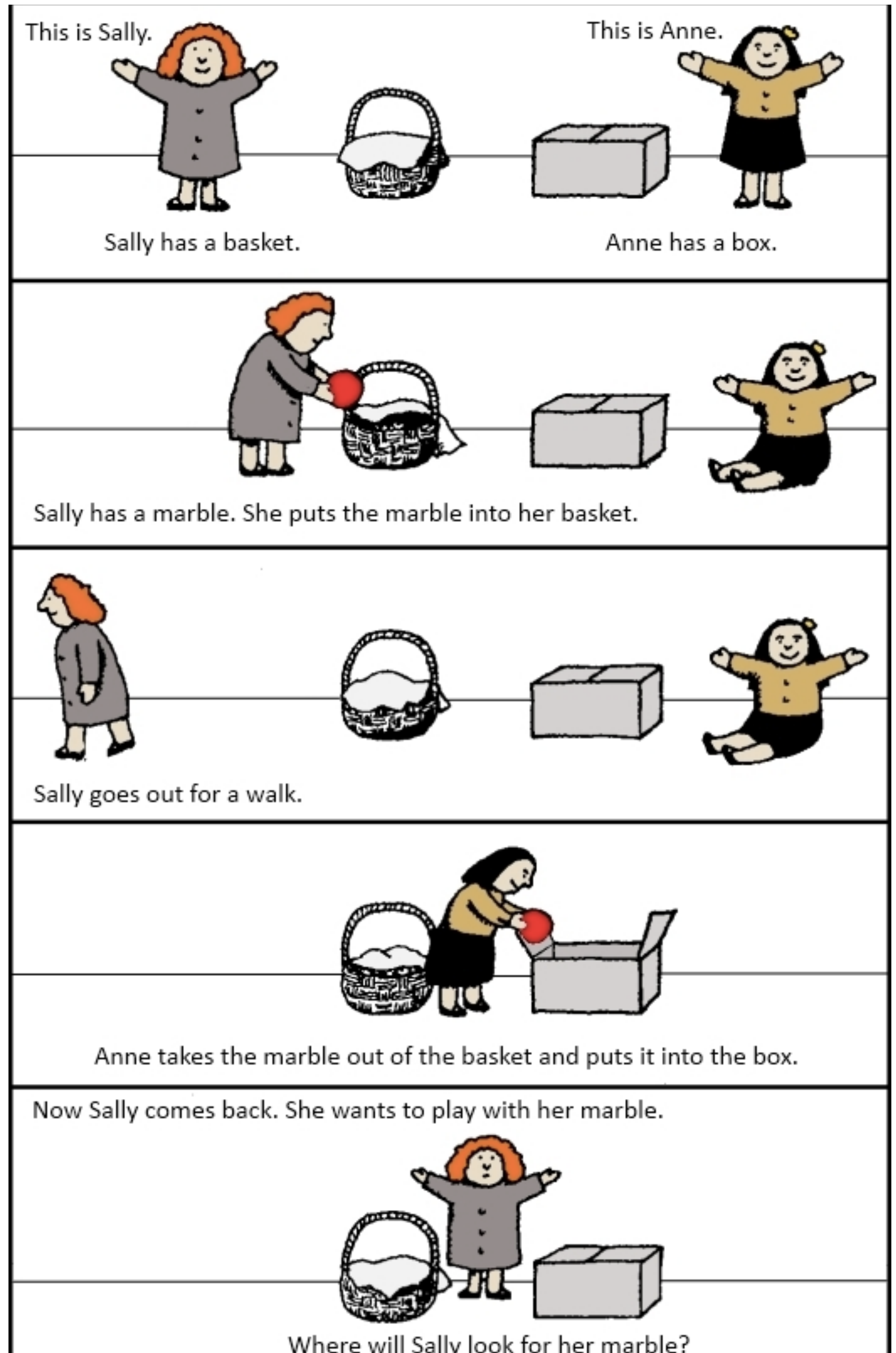
LEARNING, TEACHING, & STRATEGIES

TOPIC 4: SOCIAL COGNITION

HIGHER-ORDER REASONING

- ‘I believe that Ann knows that Ben thinks . . . ’
- Interacts with modal logic and game-theory
- Two major experimental paradigms:
  - false belief tasks
  - turn-based games

TOPIC 4.1: FALSE-BELIEF TASKS



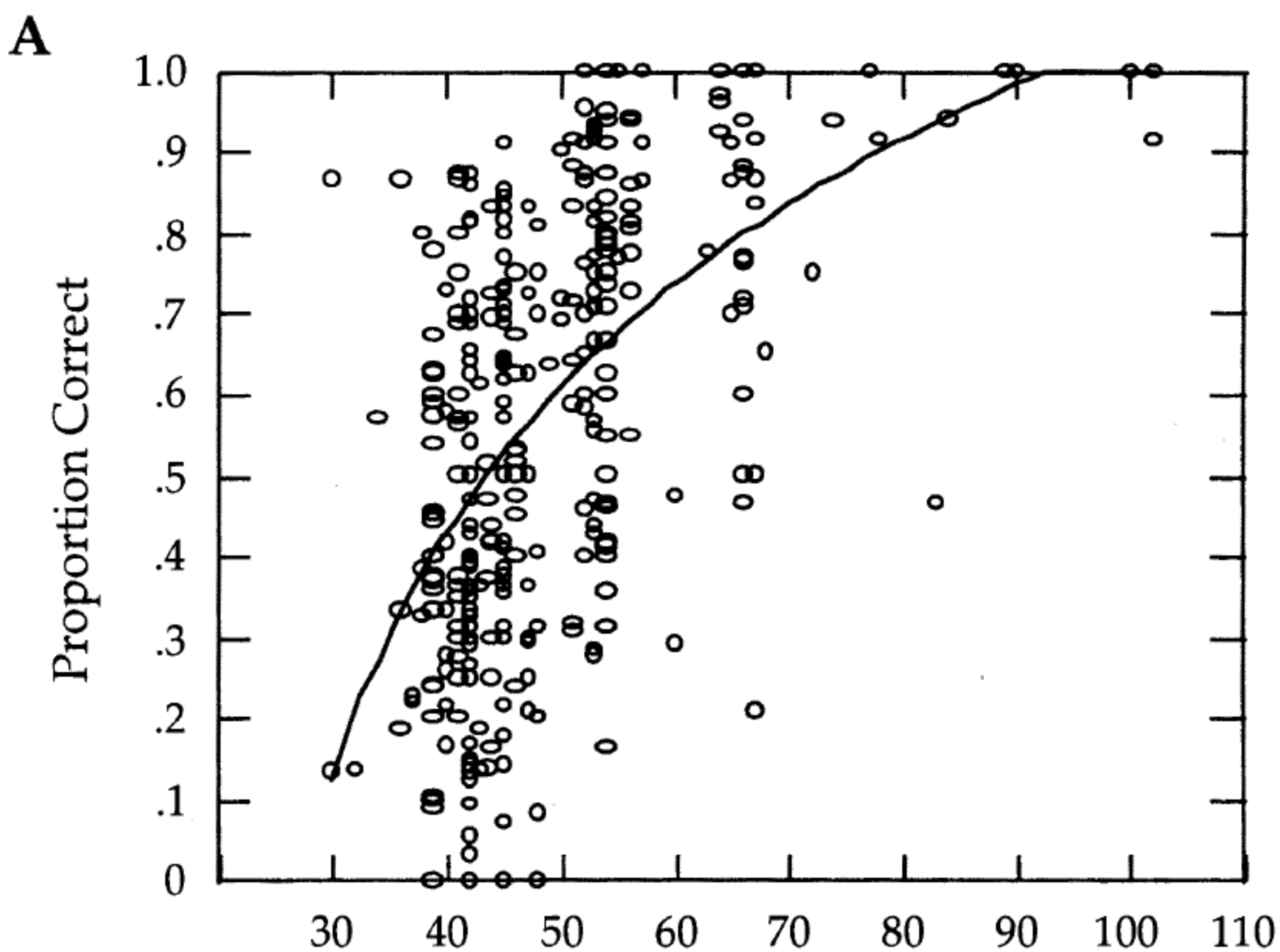
Wimer & Perner'83

Baron-Cohen et al.'85

SMARTIES TEST

1. Peter, is shown a Smarties tube
2. Smarties have been replaced by pencils
3. "What do you think is inside the tube?"
4. Peter answers: "Smarties!"
5. The tube is then shown to contain pencils only.
6. "Before it was opened, what did you think was inside?"
7. ???

DEVELOPMENT OF TOM



Wellman et al.,'01

Autistic children have a delayed ability to answer correctly

CLOSED-WORLD REASONING ANALYSIS

$$\mathit{see}_a(\varphi) \implies B_a(\varphi)$$

Principle of inertia and closed world reasoning.

Prepotent response: $B_b(\varphi) \wedge \neg ab \implies R_b(\varphi)$

In some children it can be inhibited in others not.

Van Lambalgen & Stenning'08

SO, TWO COMPETING RULES

- (#) Prepotent response: $B_b(l(i,t)) \wedge \neg ab_b \rightarrow R_b(l(i,t))$
- (*) Partial comprehension: $B_b(B_a(l(i,t))) \wedge \neg ab_b \rightarrow R_b(B_a(l(i,t)))$
- (#) inhibits (*): $R_b(l(j,t)) \rightarrow ab_b$
- (*) inhibits (#): $B_b(B_a(l(i,t))) \rightarrow ab_b$

A HYBRID LOGICAL ANALYSIS

- Reasoning is about shifting to a different perspective:
  - At the time a, Peter deduces that there are Smarties inside the tube
  - If Peter deduces ϕ then Peter believes ϕ
  - Hence, @a, Peter believes that there are Smarties inside.

$$\dfrac{@_a Ds \qquad \dfrac{[a] \qquad \dfrac{\dfrac{[a] \qquad [@_a Ds]}{Ds}\,(@E) \qquad \dfrac{}{Ds \to Bs}}{Bs}\,(\to E)}{@_a Bs}\,(@I)}{@_a Bs}\,(\mathit{Term})$$

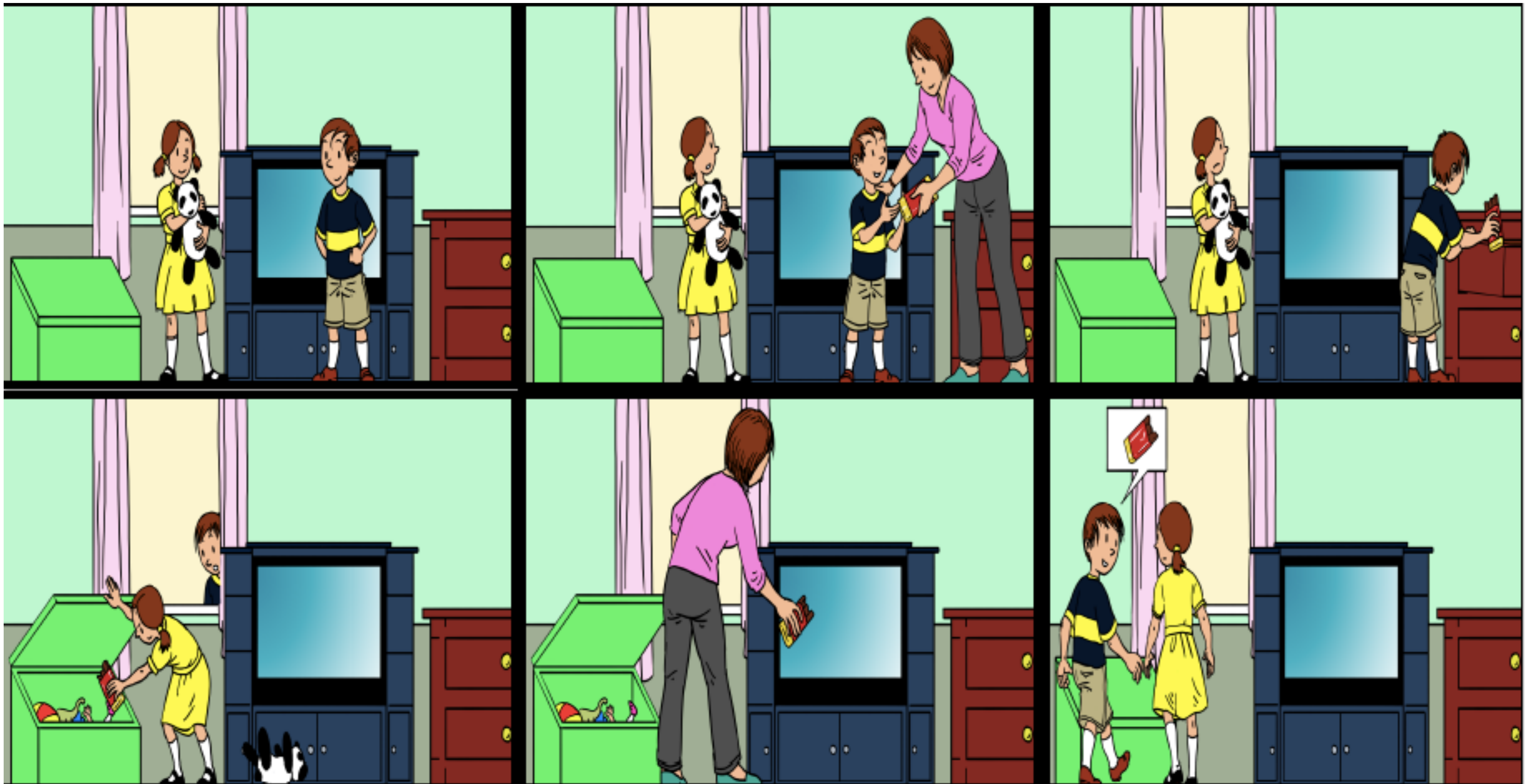
Braüner'13

WHAT’S THE CONCLUSION?

- Different diagnoses
- Do they predict differences in processing?
- How one could experimentally compare the models?
- What about erroneous reasoning?
- Do they shed any light on the developmental process?

There are other logical formalizations, e.g, Bolander 2014 uses DEL.

TOPIC 4.2: 2ND ORDER FALSE BELIEF TASK



Reality control question: Where is the chocolate now? Zero-order (TV stand)

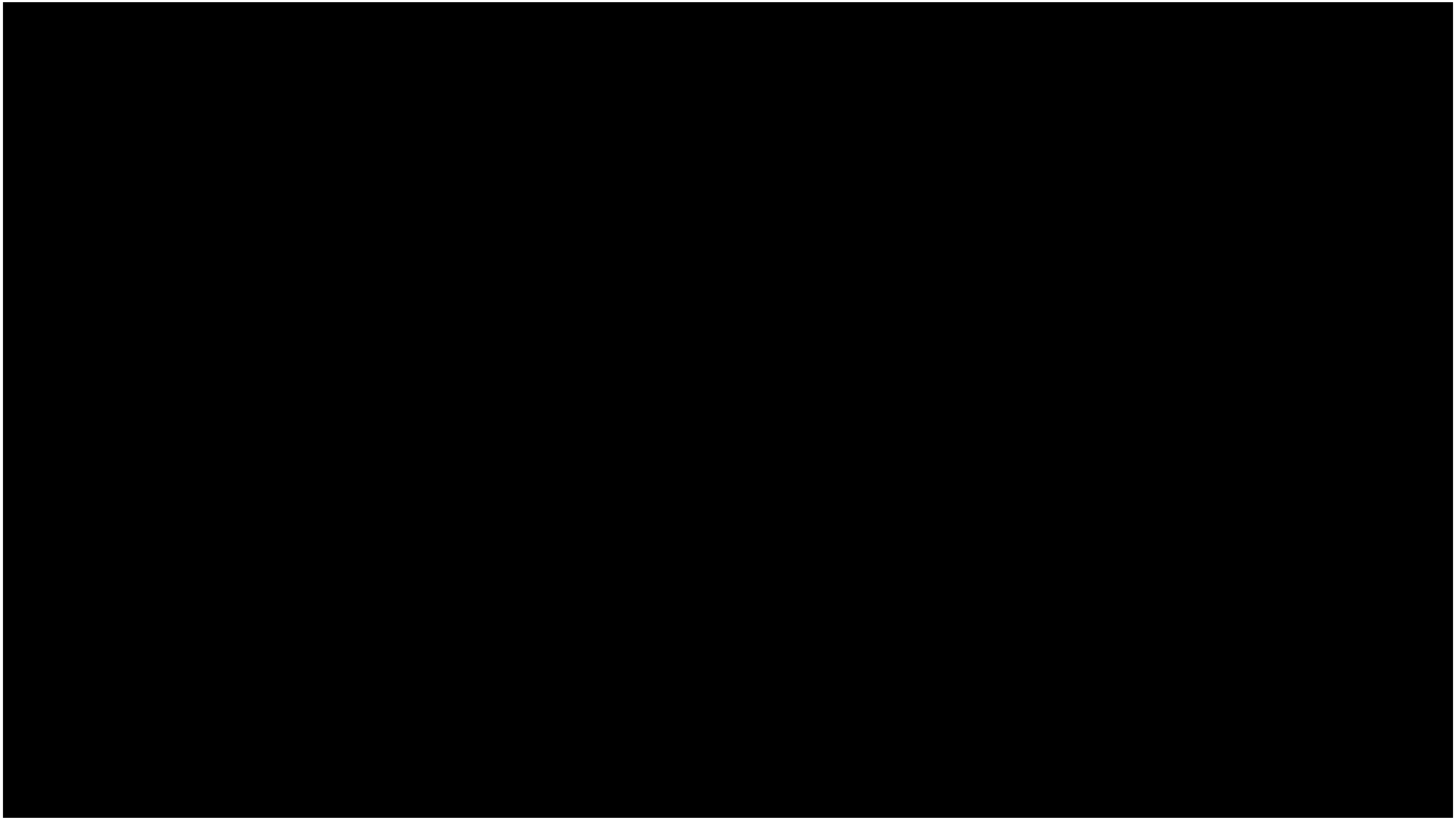
1st order false belief: Where will Murat look for the chocolate? First-order (Toy box)

2nd order false belief: Where does Ayla think that Murat will look for the chocolate?

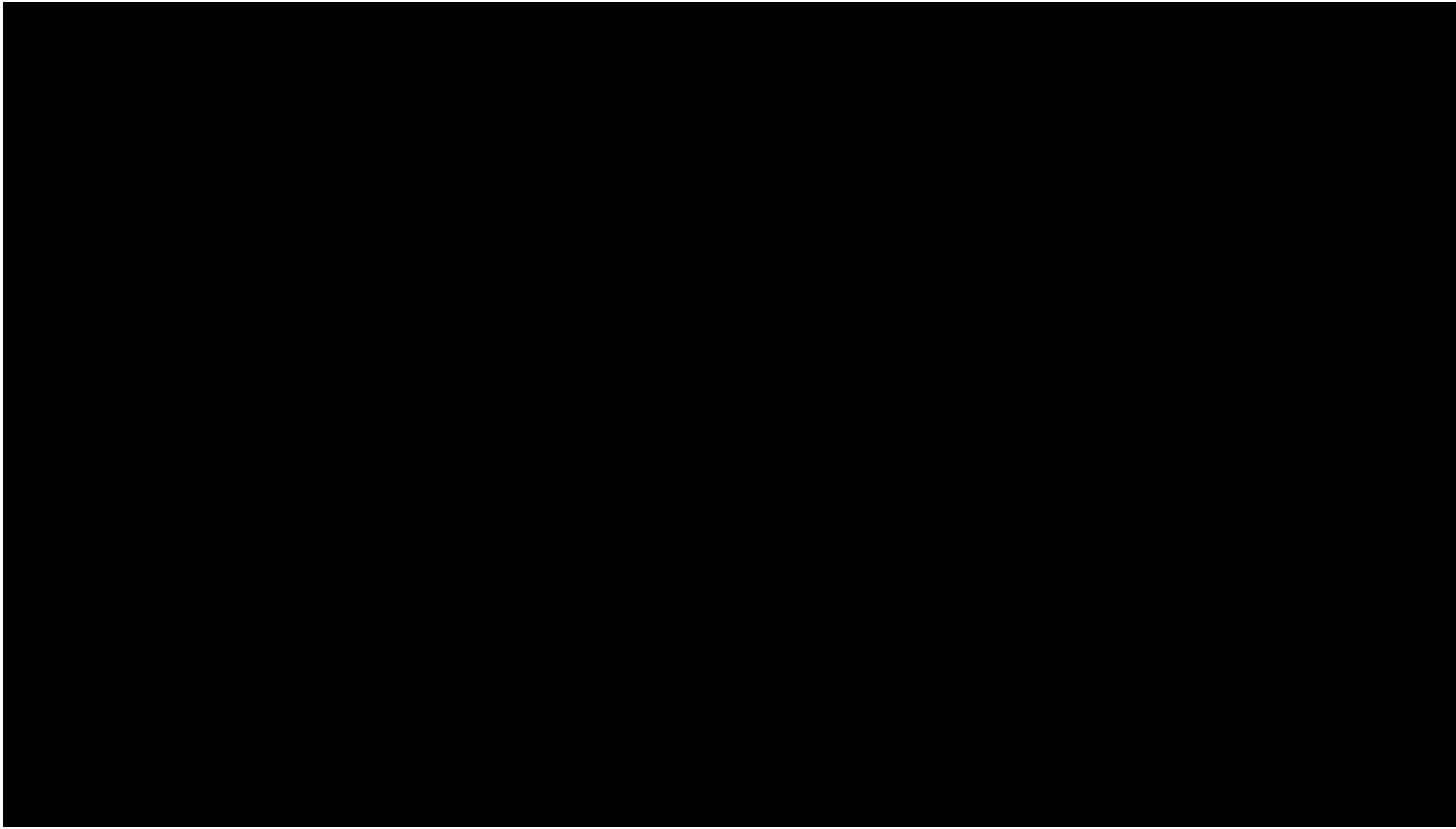
Second-order (Drawer)

Aarslan et al.'13.

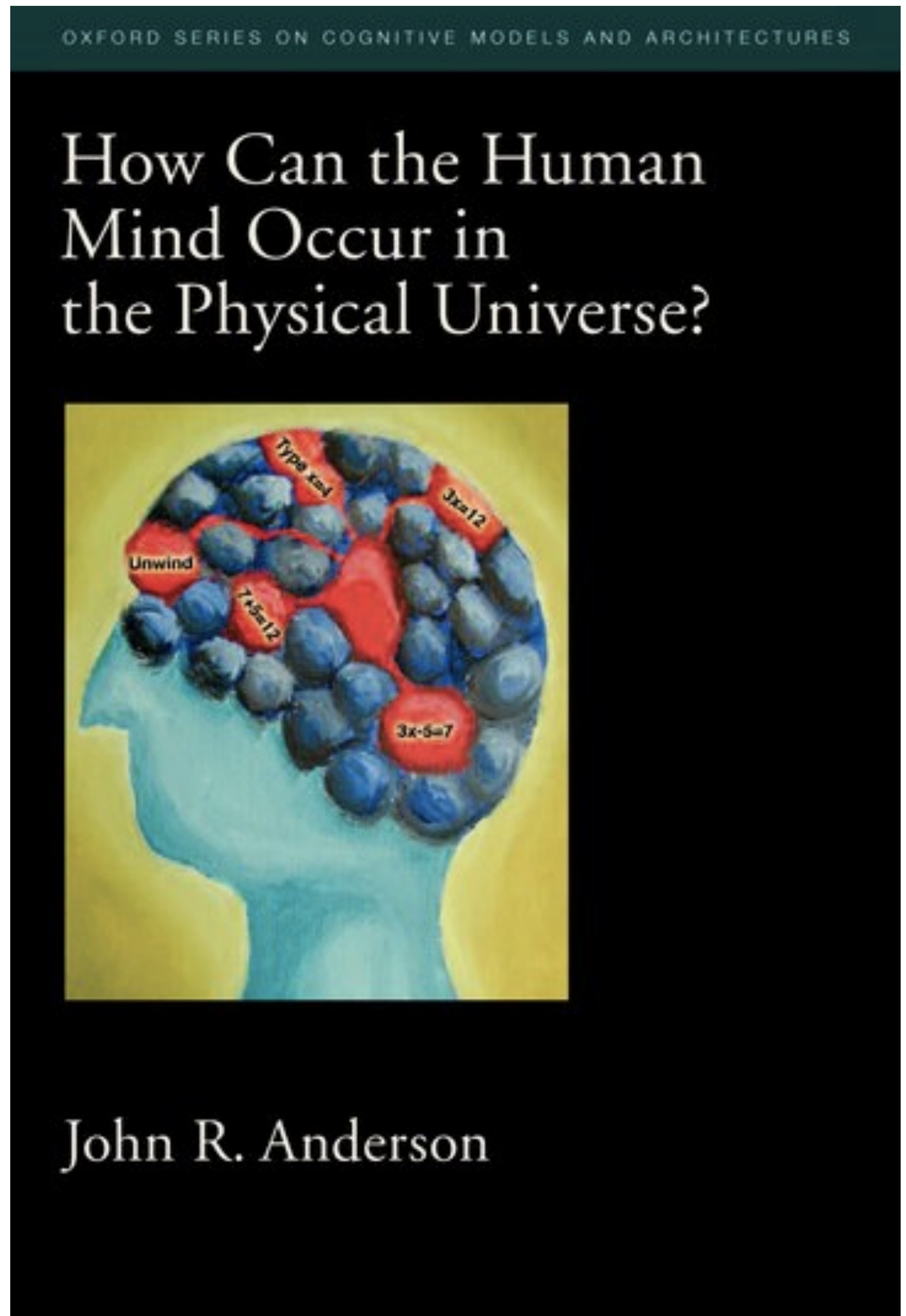
USING COGNITIVE ARCHITECTURES



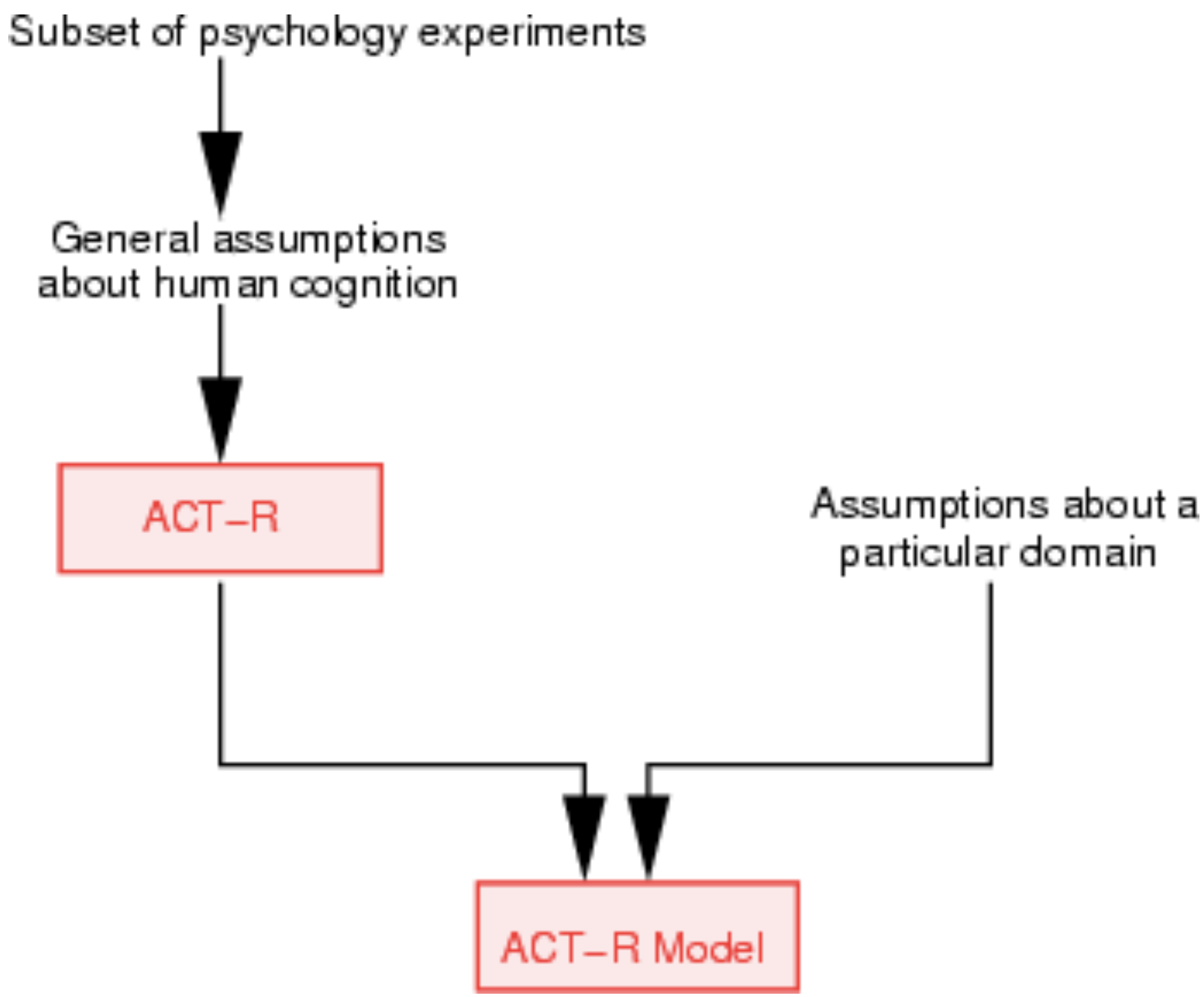
USING COGNITIVE ARCHITECTURES



ACT-R:
ADAPTIVE CONTROL OF THOUGHT, RATIONAL

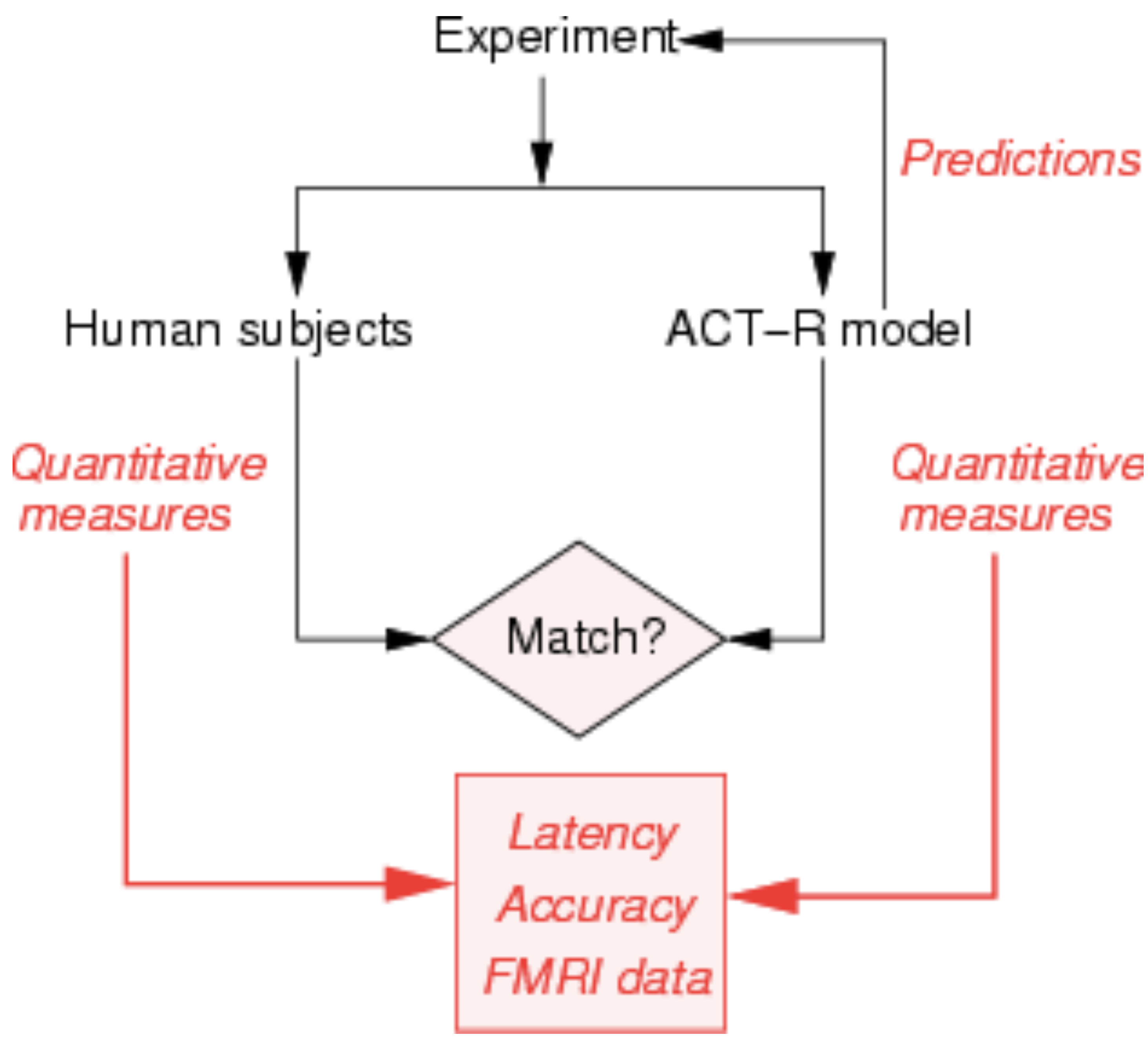


WHAT IS ACT-R?



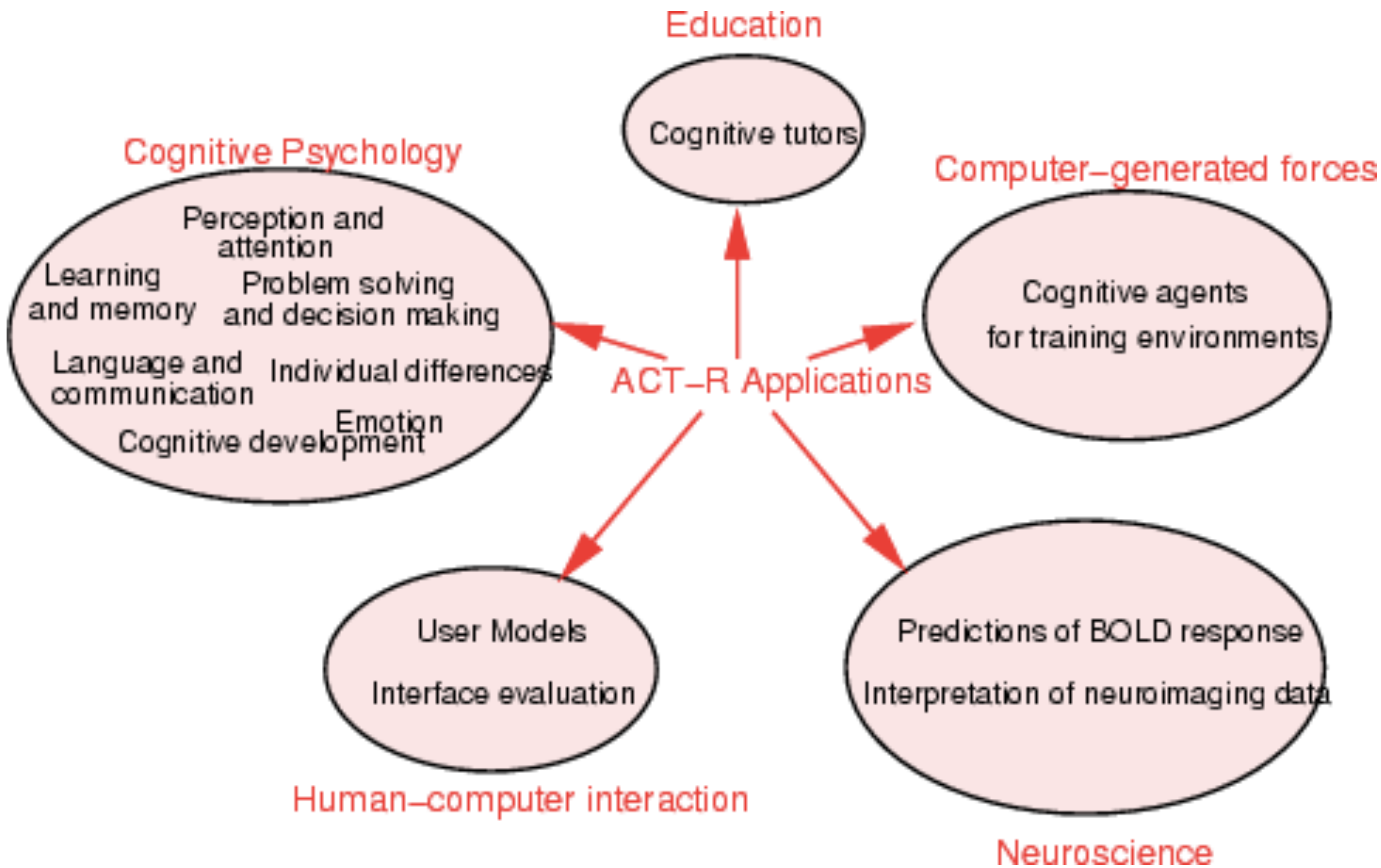
Subset of psychology experiments
General assumptions
about human cognition
ACT–R
Assumptions about a
particular domain
ACT–R Model

WHY ACT-R?

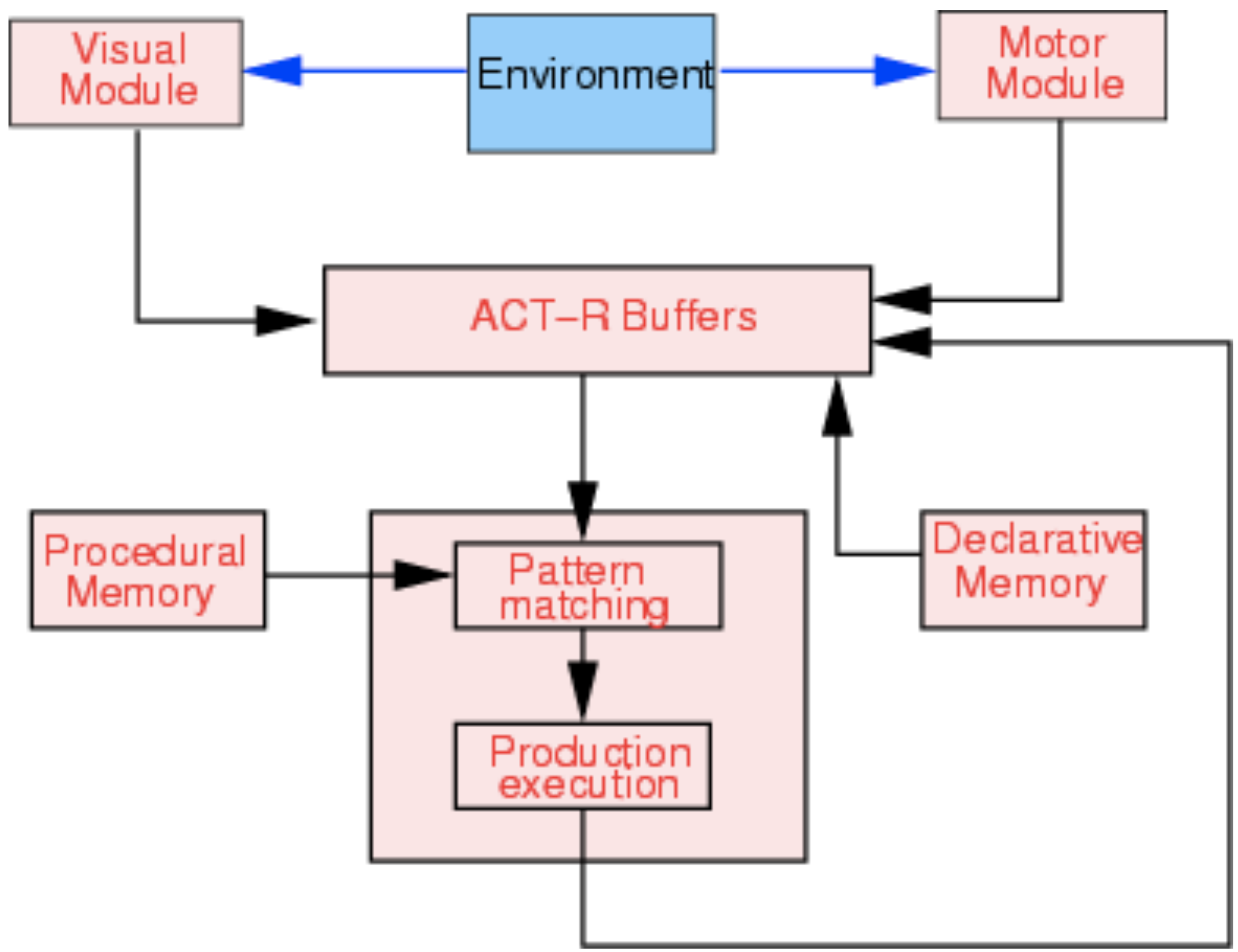


Experiment
Predictions
Human subjects
ACT–R model
Quantitative measures
Quantitative measures
Match?
Latency
Accuracy
FMRI data

APPLICATIONS



HOW DOES IT WORK?



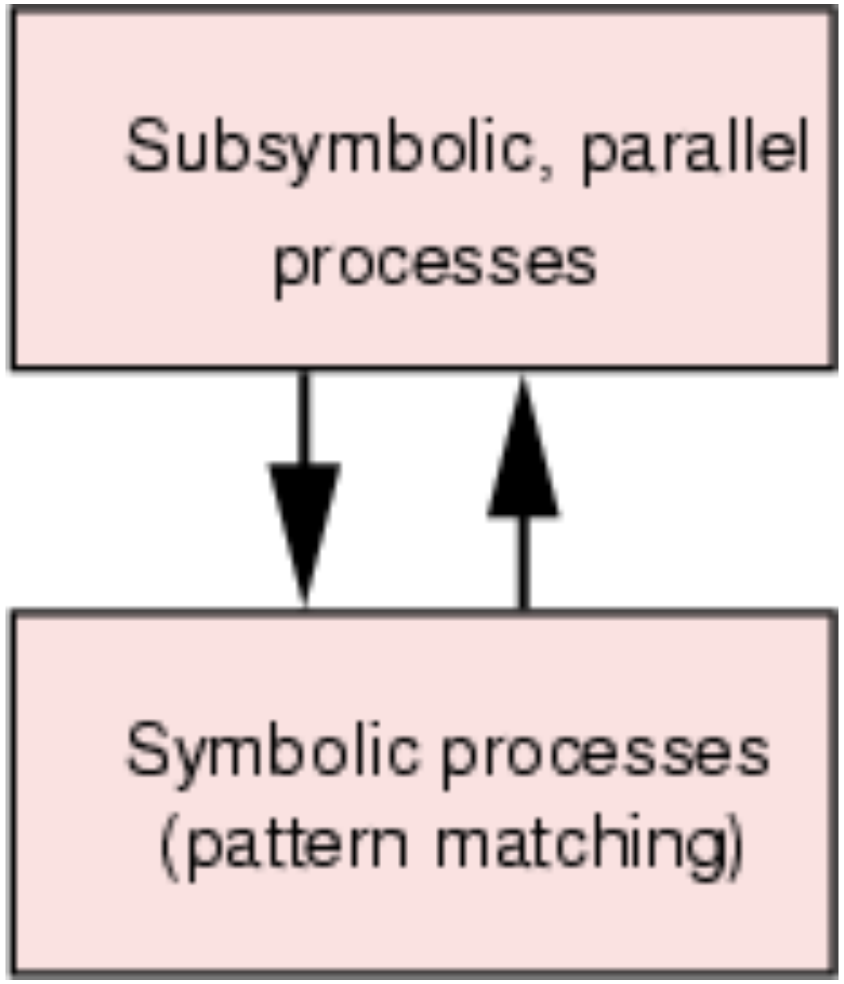
HYBRID ARCHITECTURE

- Symbolic: production system
- Sub-symbolic:

A. Utility functions for productions

B. Declarative memory retrieval

C. Learning



EXAMPLE: ACTIVATION IN DECLARATIVE MEMORY

$$A_i = B_i + \varepsilon$$

***B*$_i$:** The base-level activation. This reflects the recency and frequency of practice of the chunk i.

ε: The noise value. The noise is composed of two components: a permanent noise associated with each chunk and an instantaneous noise computed at the time of a retrieval request.

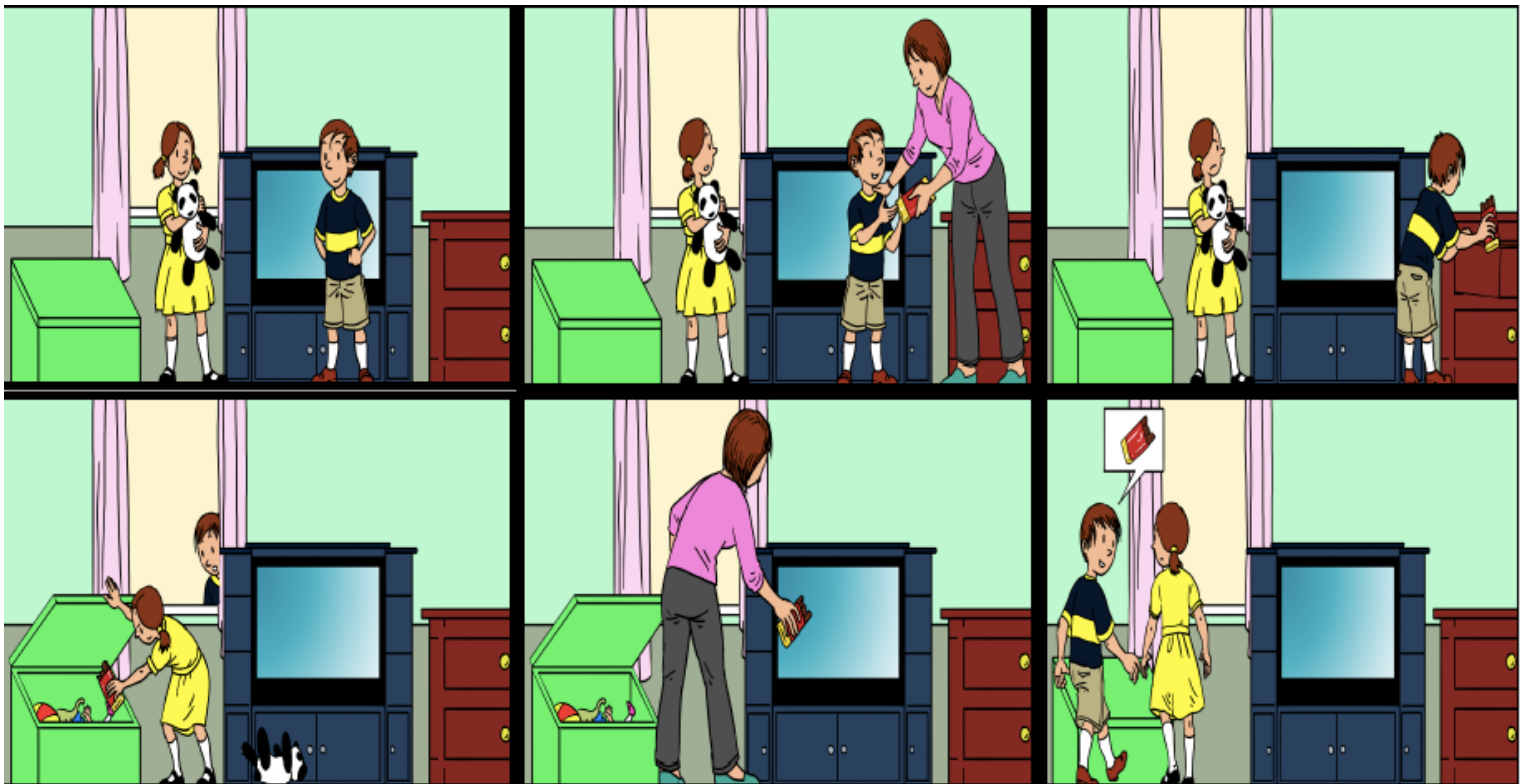
$$B_i = \ln\left(\sum_{j=1}^{n} t_j^{-d}\right)$$

***n*:** The number of presentations for chunk i.

***t*$_j$**: The time since the *jth* presentation.

d: The decay parameter which is set using the :bll (base-level learning) parameter. This parameter is almost always set to 0.5.

2ND ORDER FALSE BELIEF TASK



Reality control question: Where is the chocolate now? Zero-order (TV stand)

1st order false belief: Where will Murat look for the chocolate? First-order (Toy box)

2nd order false belief: Where does Ayla think that Murat will look for the chocolate?

Second-order (Drawer)

Aarslan et al.'13.

Where does Ayla think that Murat will look for the chocolate?

s1 s2 s5

Retrieve a story fact that has an action verb in its slots.

s5

Check the time slot of the retrieved story fact and if it is not the latest fact, request the latest one.

Reality bias

(reasoning level 0)
(reasoning level 1)
(reasoning level 2)

Request a retrieval of one of the reasoning levels from declarative memory.

If the kth-order reasoning is retrieved, determine whose knowledge the question is about and give an answer accordingly.

WRONG!
CORRECT!

Based on the feedback, the model will strengthen successful strategy chunks, or will add or strengthen an alternative strategy if the current one failed.

(reasoning level 1)
(reasoning level 2)

Declarative Memory

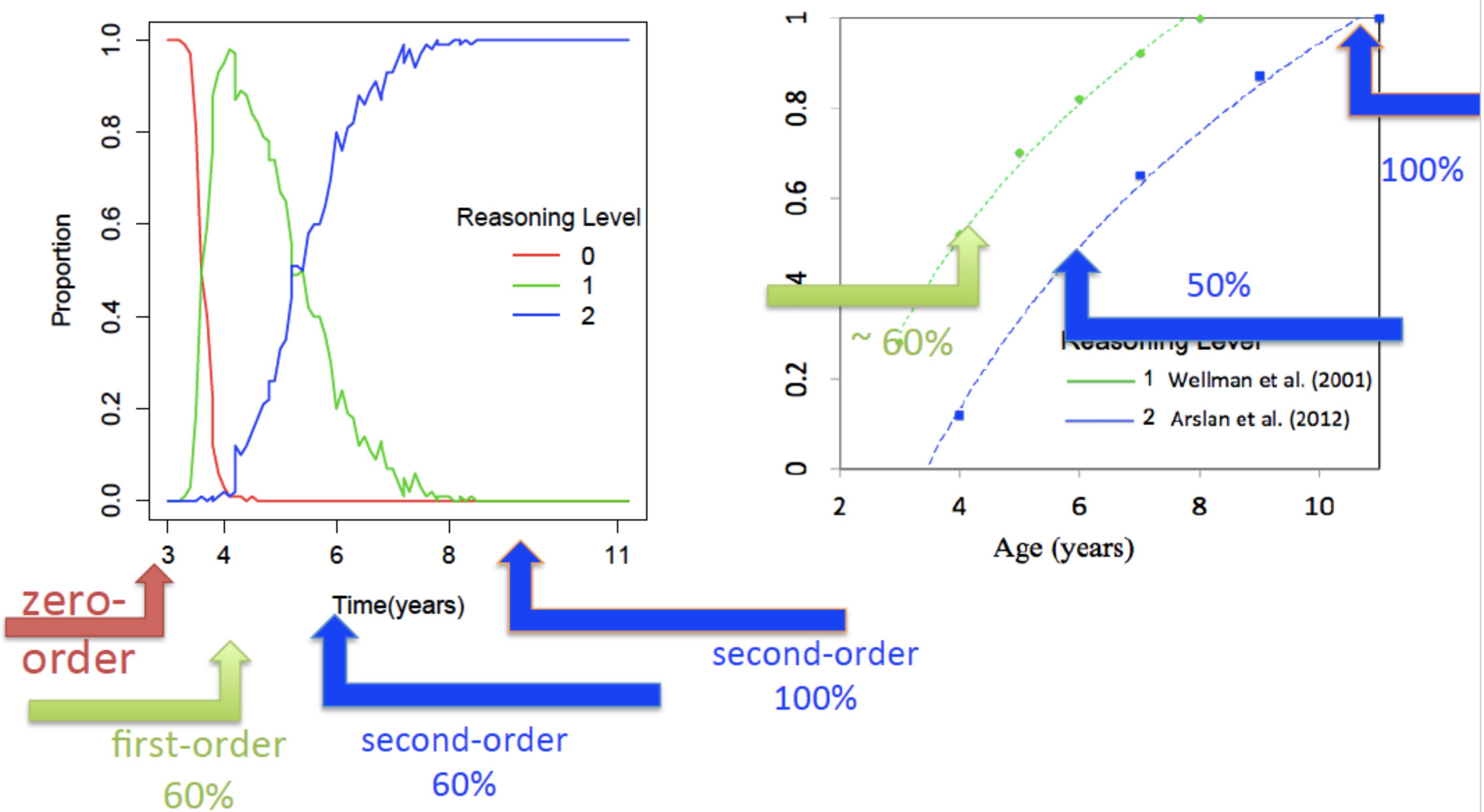
(s1 murat put chocolate drawer time 1 type action self-reference s1)
(s2 ayla put chocolate toybox time 2 type action self-reference s2)
(s3 murat see time 2 perception self-reference s3 reference s2)
(s4 ayla did not see time 2 type perception self-reference s4 reference s3)
(s5 mother put chocolate tvstand time 3 type action self-reference s5)

(reasoning level 0) (reasoning level 1) (reasoning level 2)

Aarslan et al.'13.

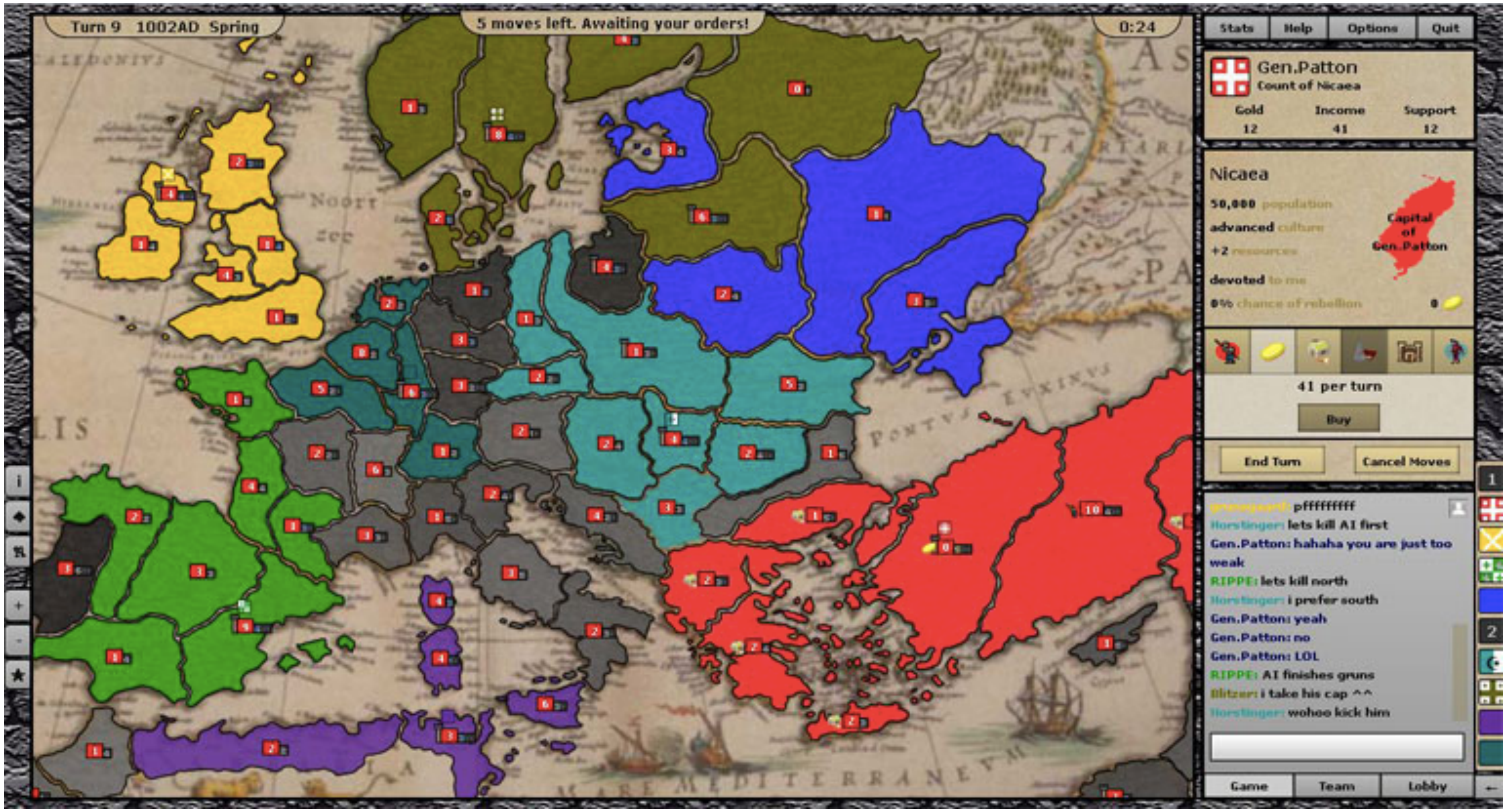
RESULTS

To average the results across 100 children, we made 100 simulations. Thus, we ran the model 10,000 times in total.

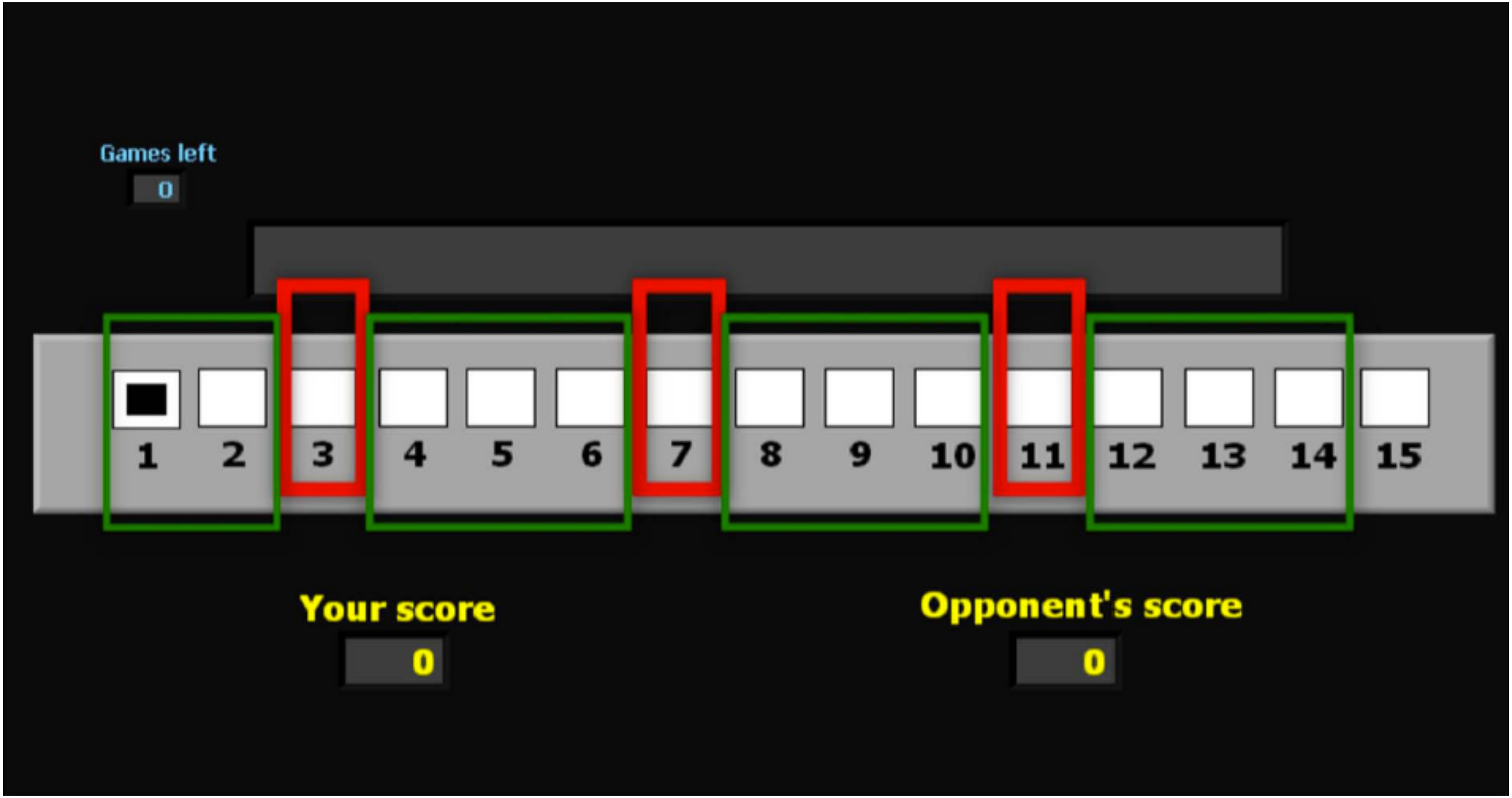


Aarslan et al.'13.

TOPIC 4.3 TURN-BASED GAMES



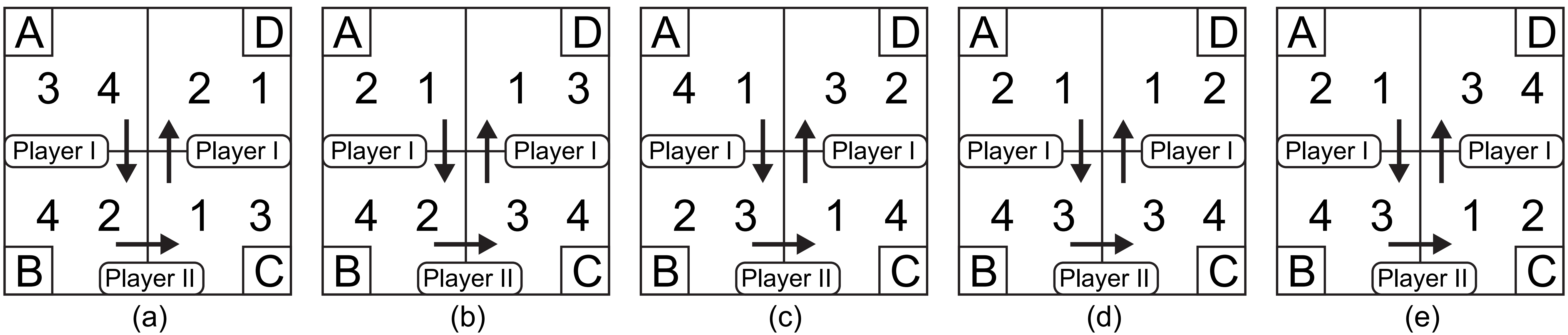
HIT-N GAME



Gneezy et al.'10

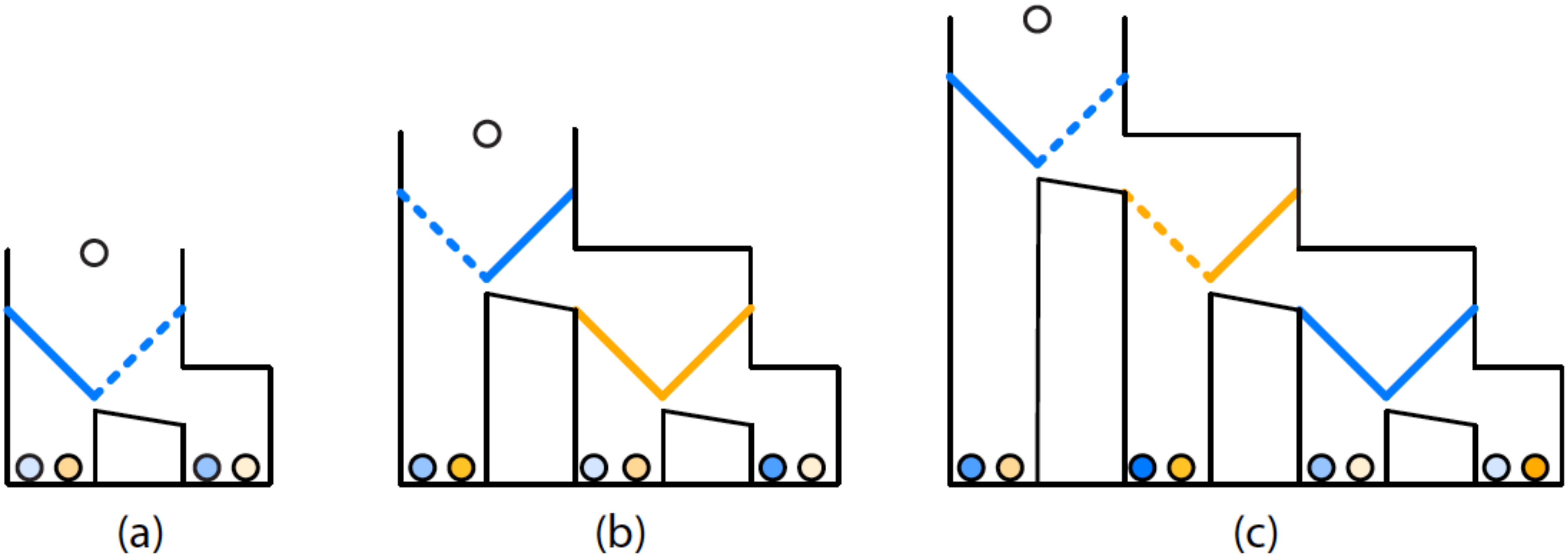
Hawes et al.'10

MATRIX GAMES



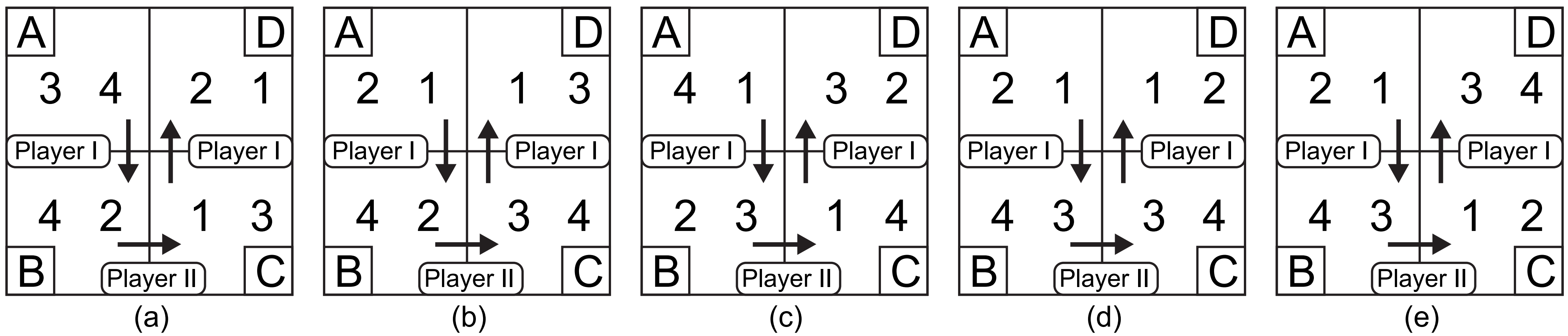
Hedden & Zhang'02

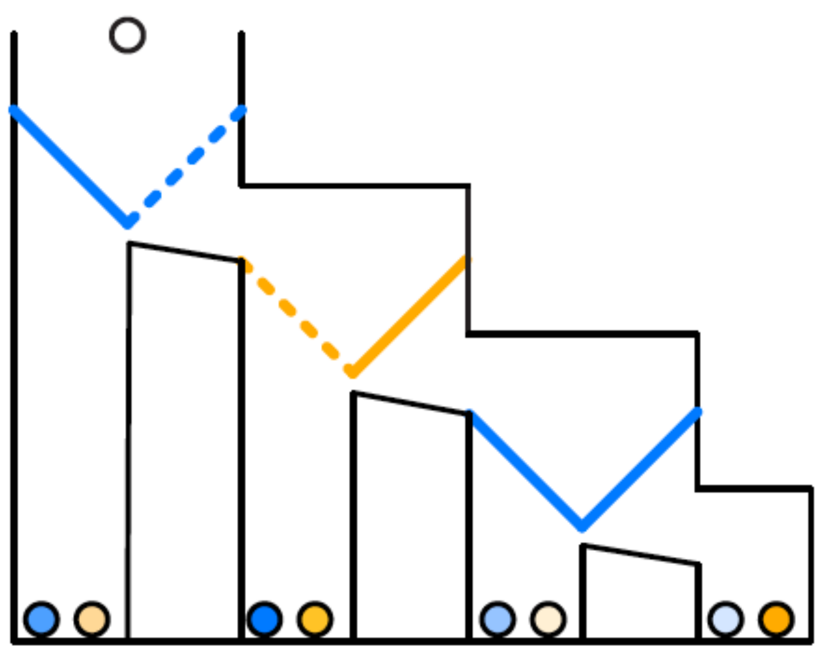
MARBLE DROP GAME



Meijering et al.'10

LOGICAL EQUIVALENCE BUT DIFFERENT BEHAVIOR



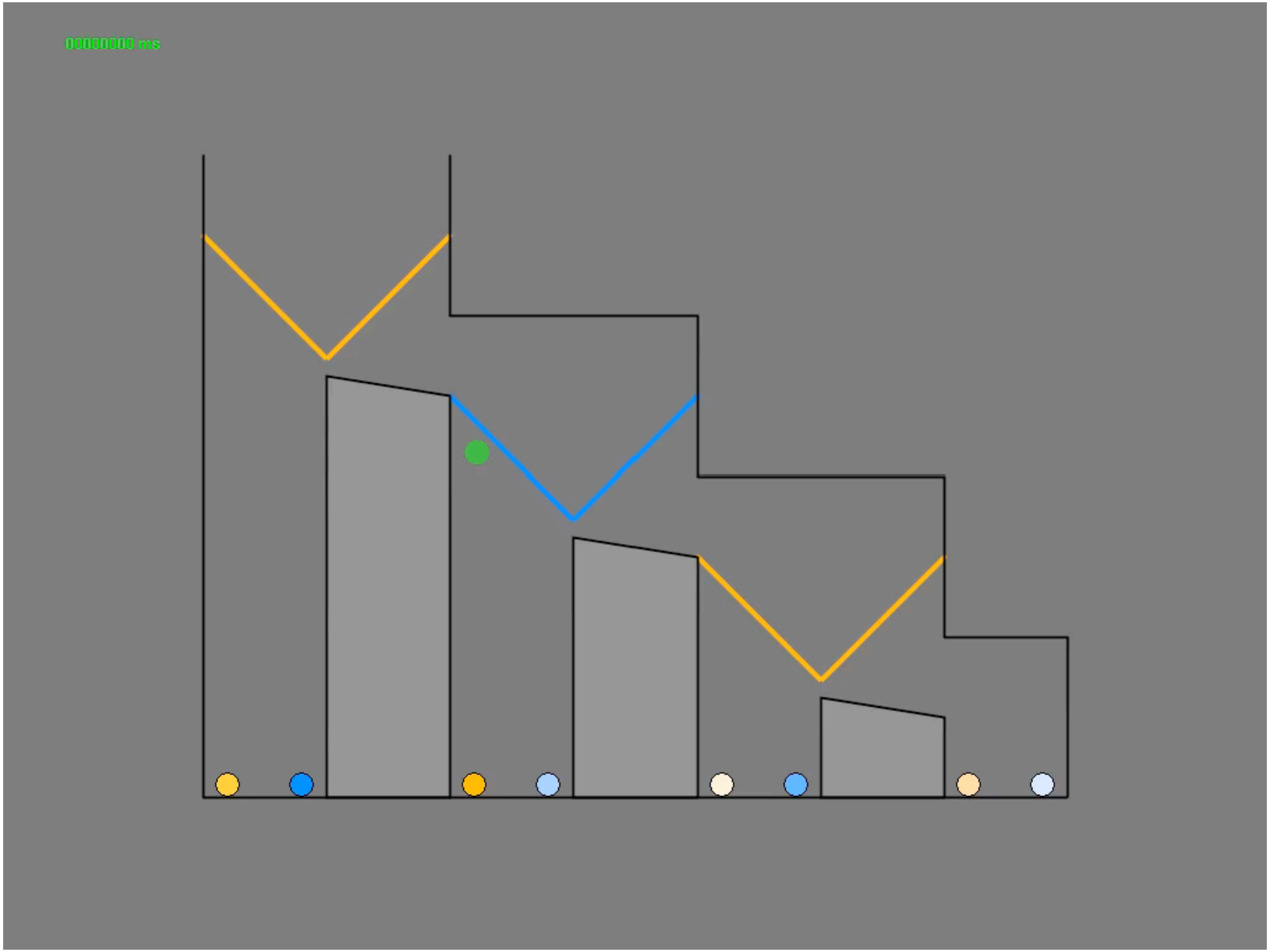


Meijering et al.'10

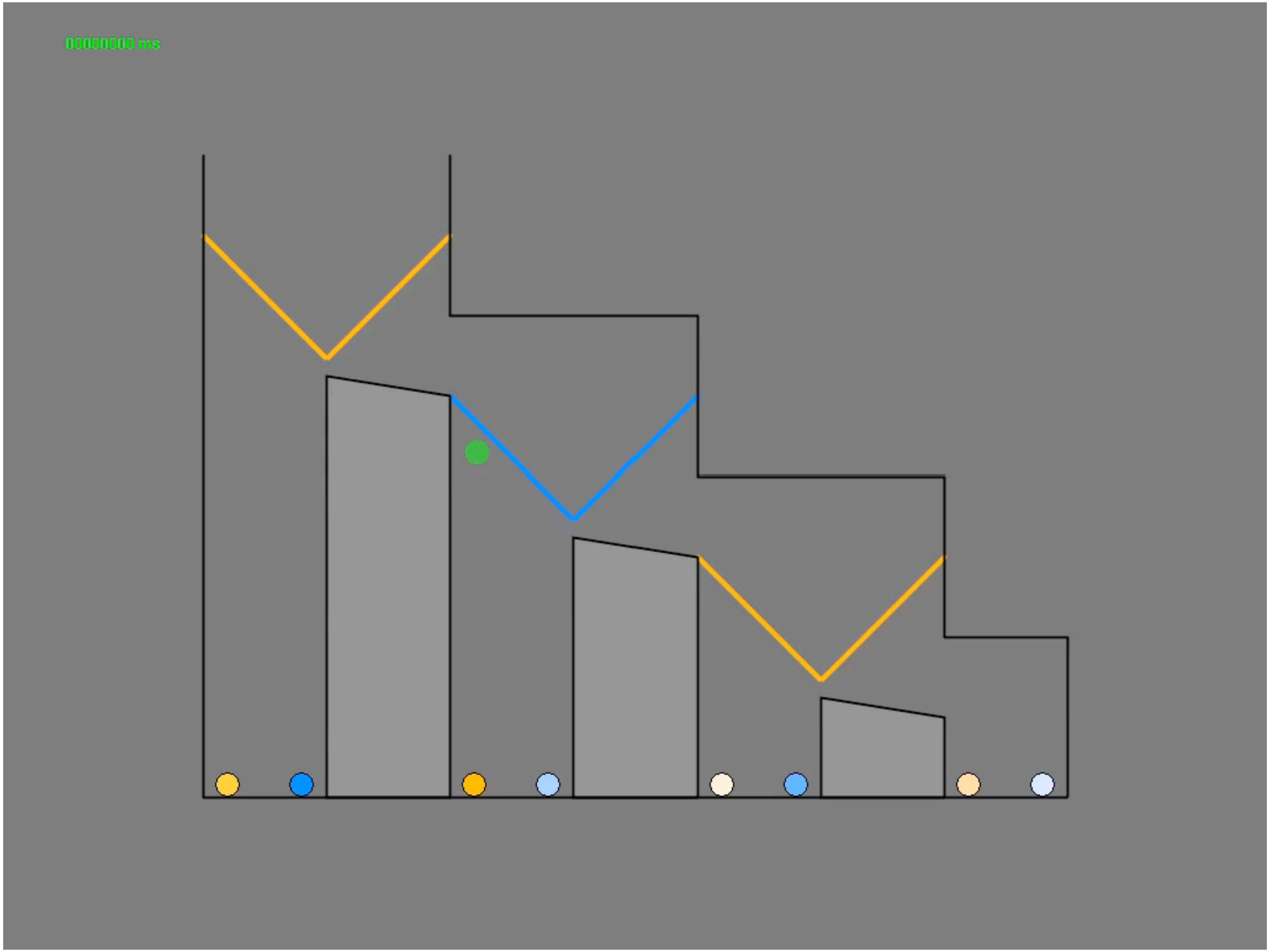
DO PEOPLE PLAY BACKWARD INDUCTION?

At the end of the game, players have their values marked. At the intermediate stages, once all follow-up stages are marked, the player to move gets her maximal value that she can reach, while the other, non-active player gets his value in that stage.

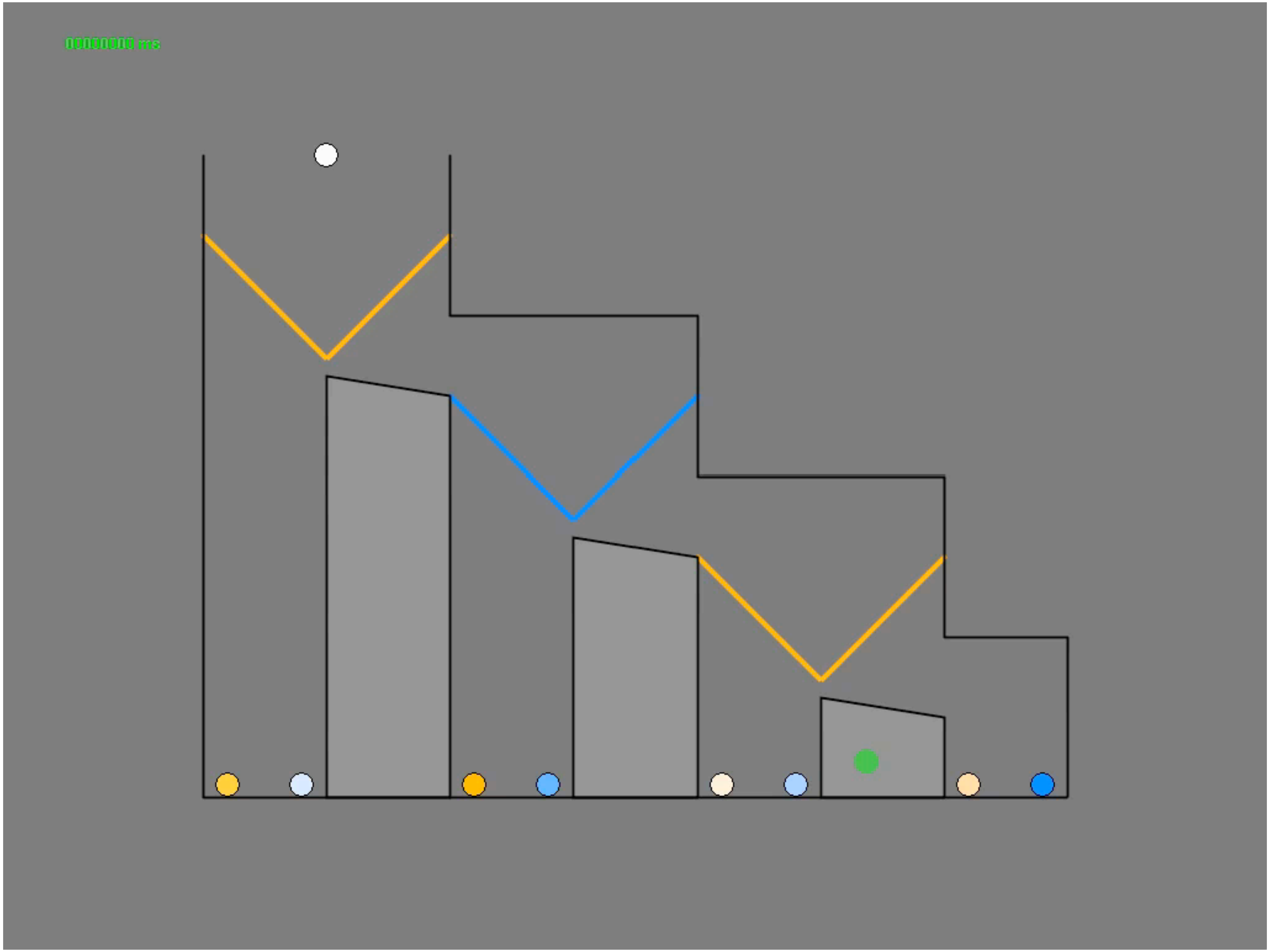
PERFORMANCE AT MDG



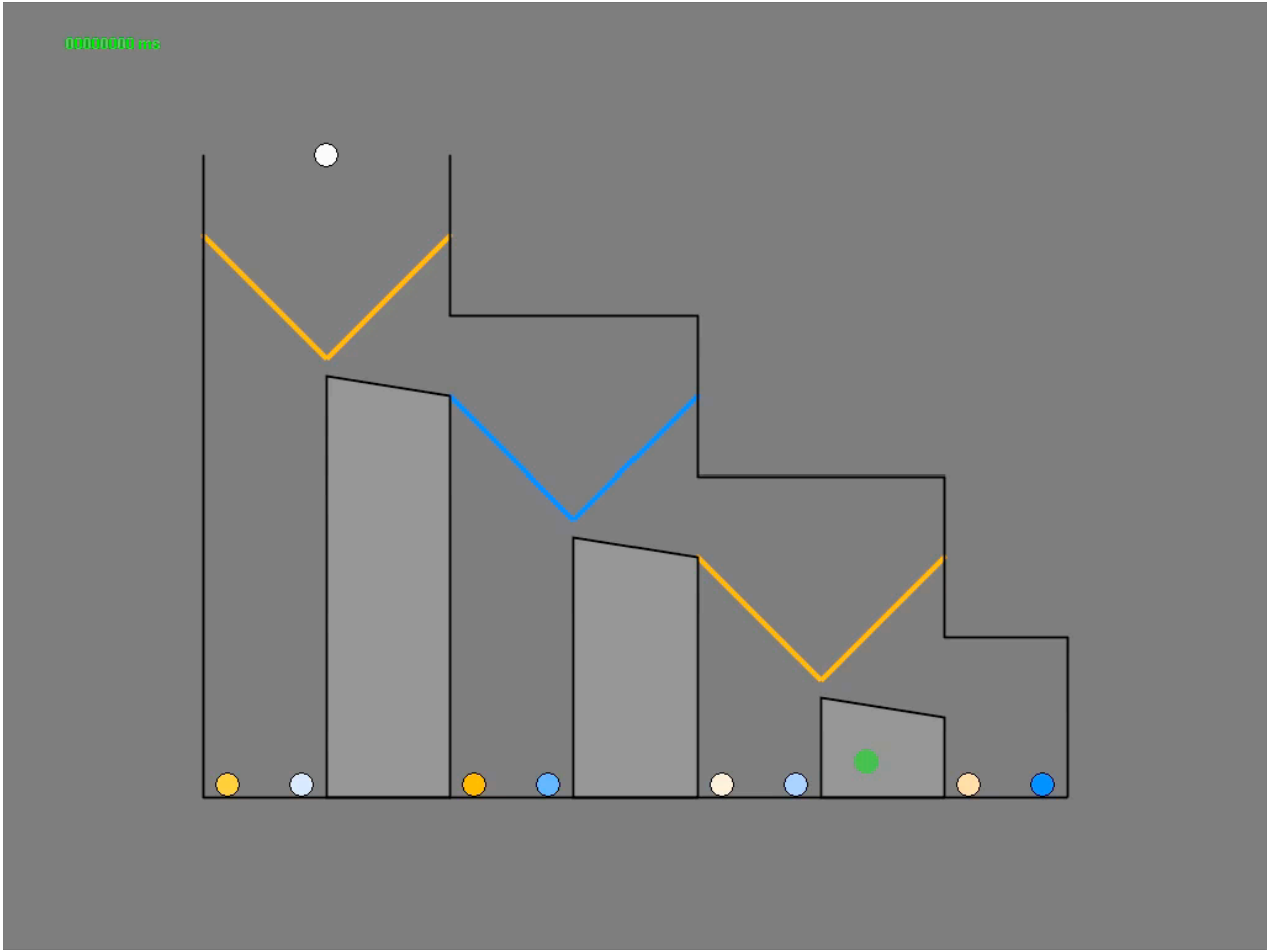
PERFORMANCE AT MDG



PERFORMANCE GETS BETTER



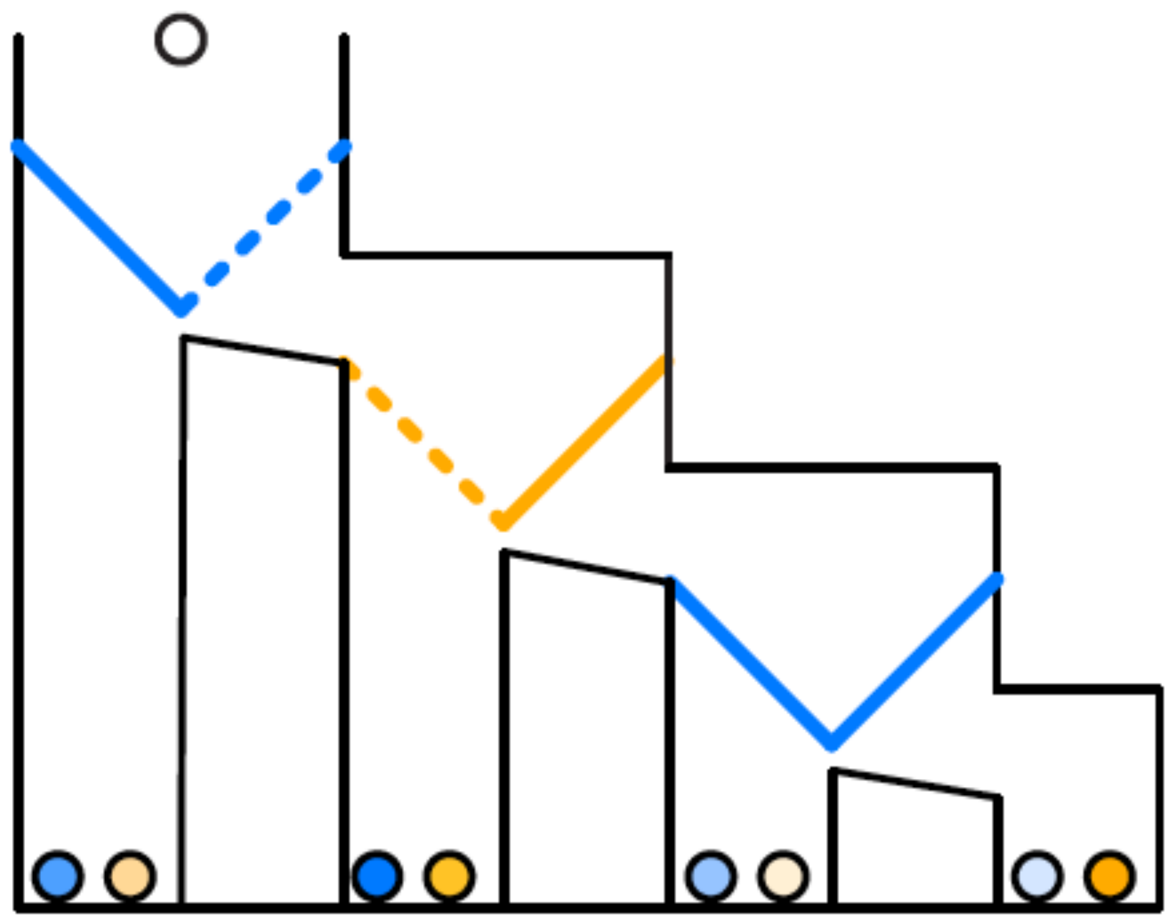
PERFORMANCE GETS BETTER



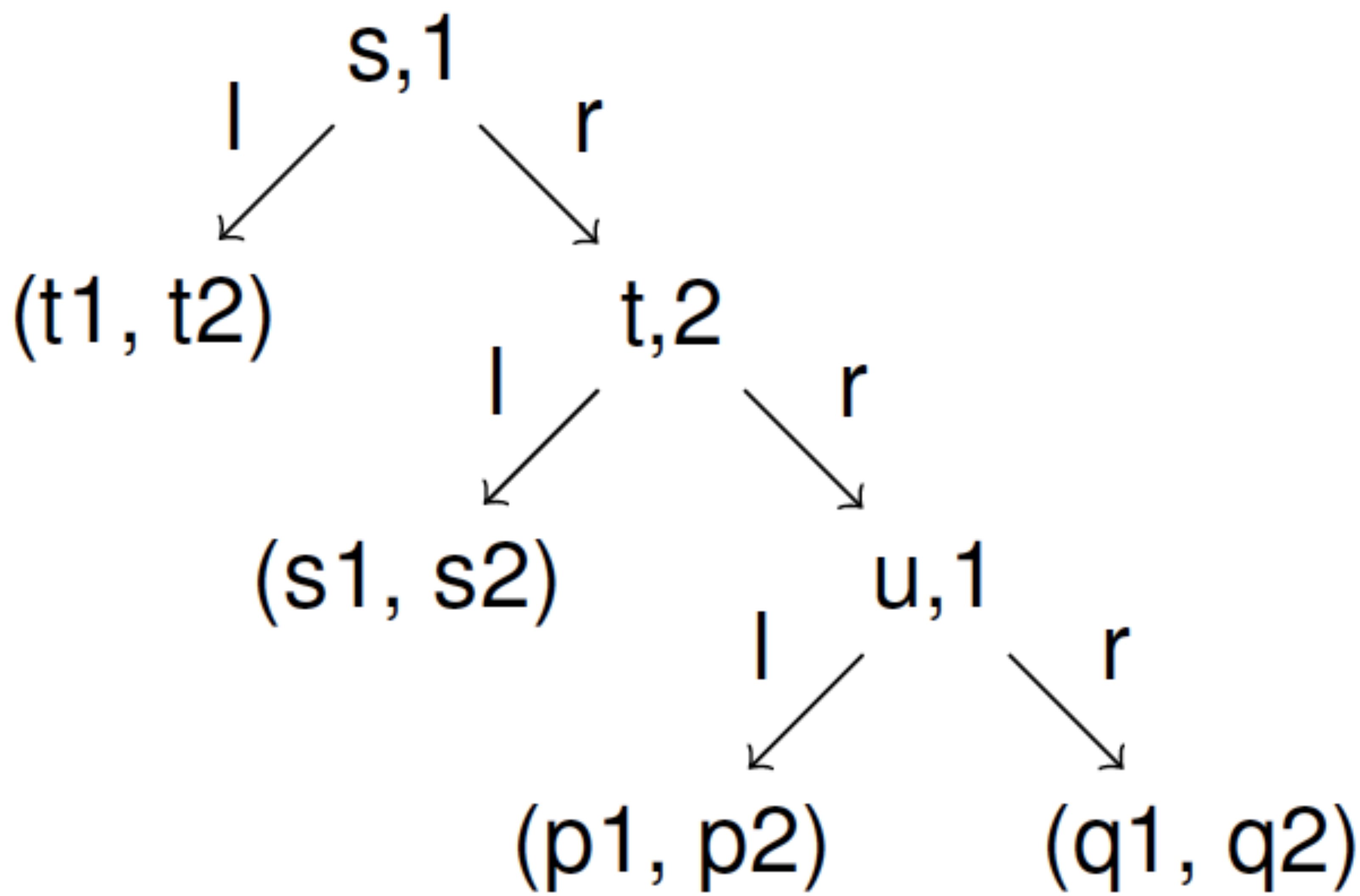
QUESTIONS

- What's going on?
- What are the cognitively important structural properties?
- How to approximate cognitive complexity?

MARBLE DROP GAME



MARBLE DROP GAME



ALTERNATION TYPE ~ LEVELS OF REASONING

Definition

Let's assume that the players strictly alternate in the game. Then:

1. In a Λ_1^i tree all the nodes are controlled by Player i.
2. In a Λ_k^i tree, k-alternations, starts with an ith Player node.

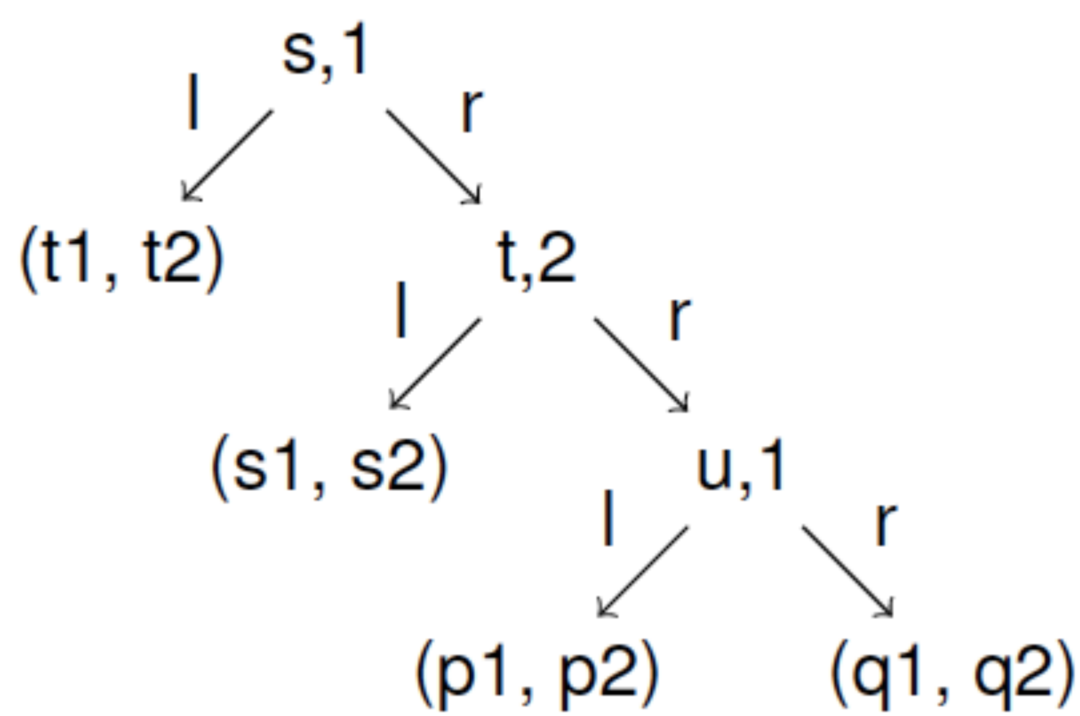


Figure : Λ_3^1 -tree

PAY-OFF STRUCTURES

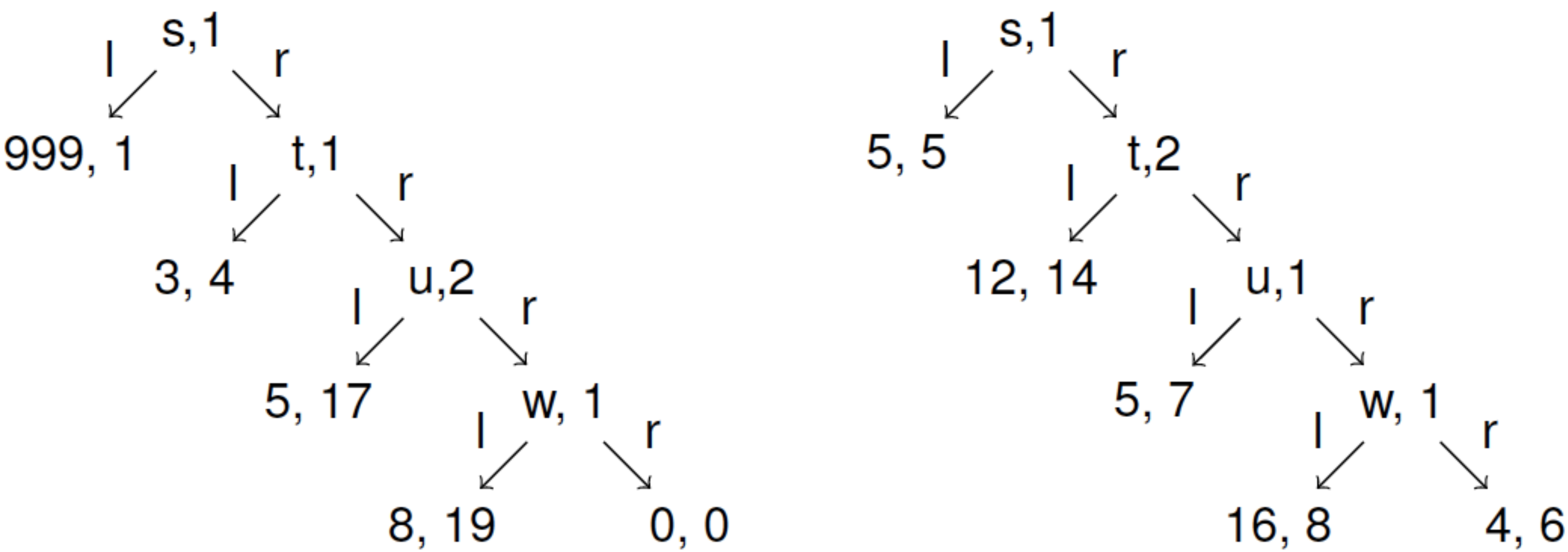


Figure : Two Λ^1_3 trees.

PAY-OFF STRUCTURES

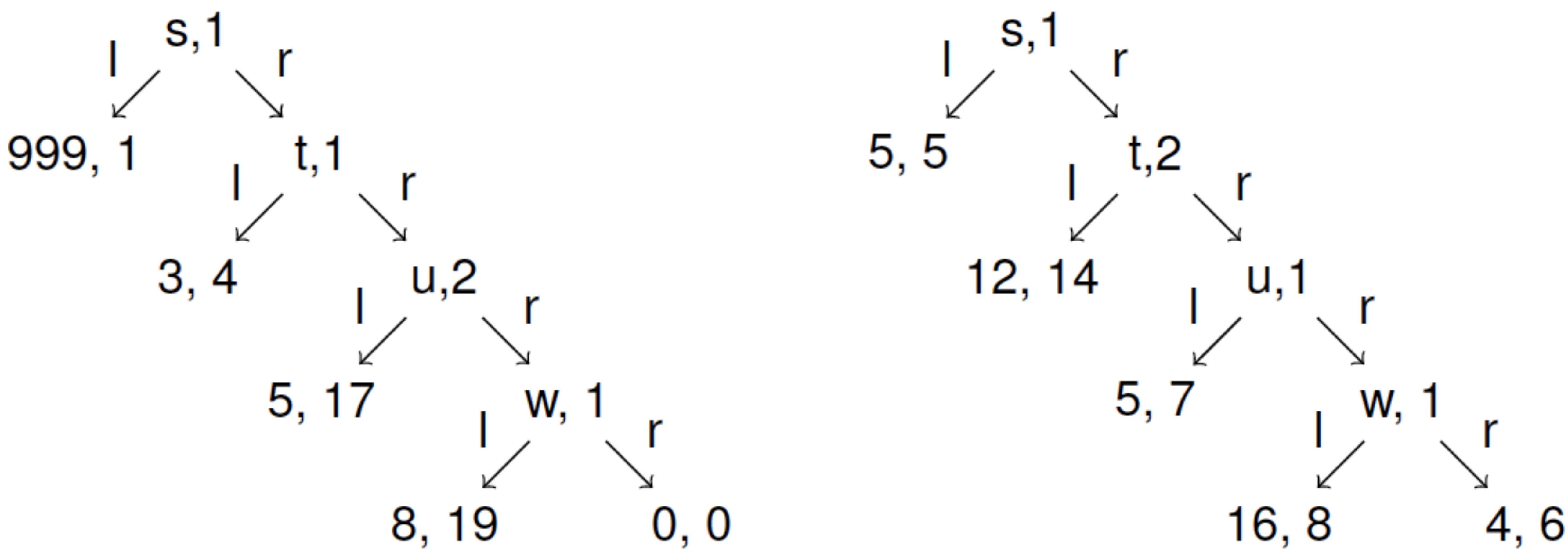
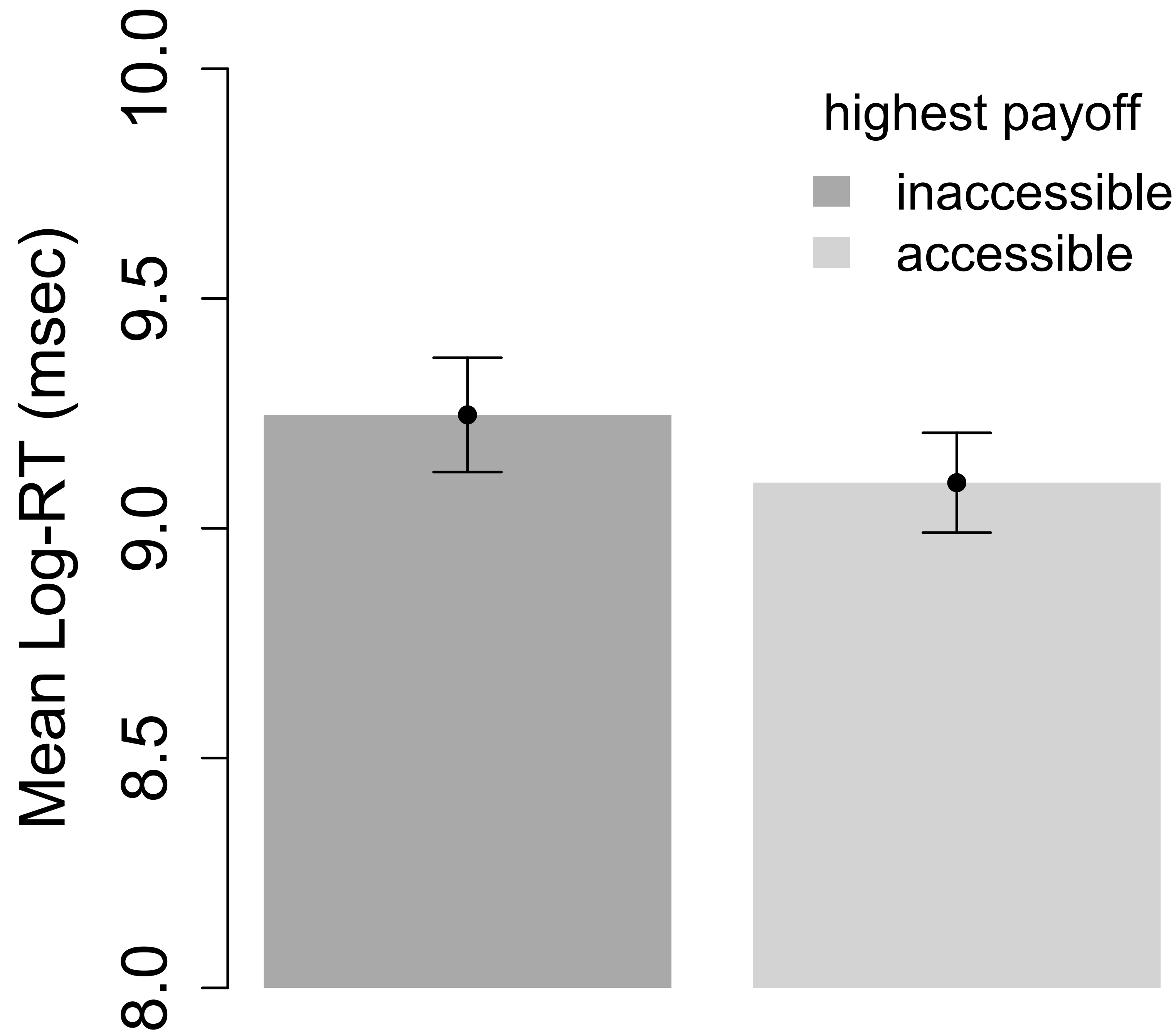


Figure : Two Λ^1_3 trees.

Forward reasoning + backtracking as an algorithmic model of the task

ACCESSIBLE VS. NON-ACCESSIBLE



Szymanik et al.'13,

SIMULATING THE ALGORITHM

Hypothesis

For an average random game with 3 decision points, the forward reasoning plus backtracking algorithm needs fewer computation steps to yield a correct solution than backward induction.

Table : Cross-table of payoff structures and the necessary number of steps when using forward reasoning with backtracking on all 576 possible experimental pay-off structures.

# of steps	1	2	4	5	6	8
# of payoff structures	288	72	48	56	16	96

On average: BI=6 and FRB=3

(Szymanik et al. 2013, CogSci)

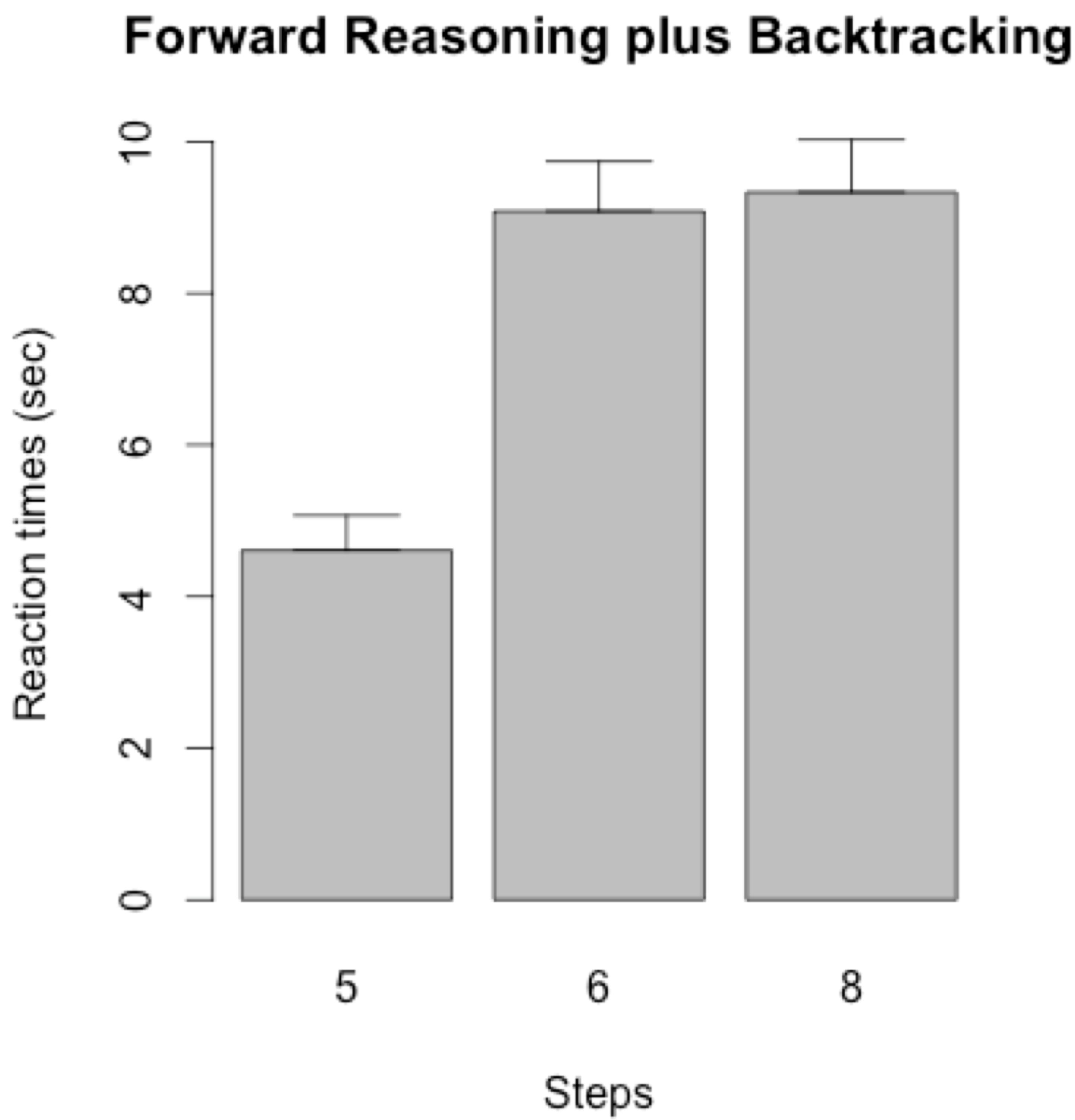
FINALLY, FRB FITS EYE-TRACKING DATA

Meijering et al.'12

FINALLY, FRB FITS EYE-TRACKING DATA

Meijering et al.'12

FRB PREDICTS RT



Szymanik et al.'13,

WHY FRB?

- FRB avoids higher-order reasoning
- Heuristics to avoid higher-order reasoning
- Normative question: Which are good?
- Descriptive question: Which are used?
- Strategic ability as a collection of algorithms.

IS THERE A TRANSFER BETWEEN FALSE BELIEF TASK AND GAMES?

Are the two task tapping into the same cognitive skills?

COURSE SUMMARY & DISCUSSION

- Logic helps to rigorously formulate cognitive problems
- Logic informs about intrinsic properties of a problem
- Often inspires cognitive models and experimental paradigms
- Which logic to choose? Only one logic of a task?
- What about learning/developmental perspective?
- How to go beyond normativity, accounting for errors?
- How to go from descriptive to processing perspective?

Take home message:

Logic informs about intrinsic properties of a problem